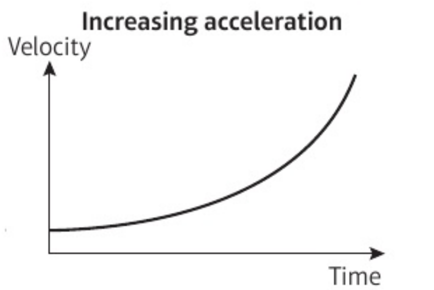
AS Mechanics – Chapter 11 – Variable Acceleration

Functions of Time

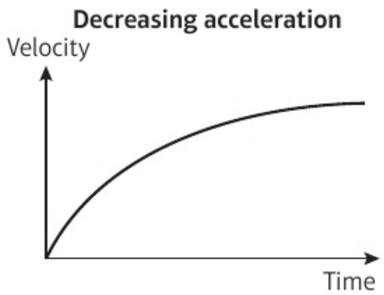
If acceleration of a moving particle is variable, it changes with time and can be expressed as a function of time.

In the same way, velocity and displacement can also be expressed as functions of time.

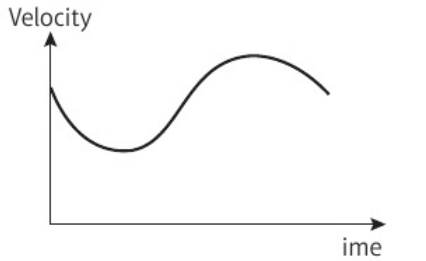
These velocity-time graphs represent the motion of a particle travelling in a straight line:



The rate of increase of velocity is increasing with time and the gradient of the curve is increasing.



The rate of increase of velocity is decreasing with time and the gradient of the curve is decreasing.



At the two stationary points on the curve, the acceleration is momentarily zero.

Rates of Change

Velocity is the rate of change of displacement.

If the displacement, s, is expressed as a function of t, then the velocity, v, can be expressed as

$$v = \frac{ds}{dt}$$

Acceleration is the rate of change of velocity.

If the velocity, v, is expressed as a function of t, then the acceleration, a, can be expressed as

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

You can use calculus to determine **local** maximum and minimum values of displacement, velocity and acceleration.

Be careful: the **greatest speed** of object may be a local *maximum* velocity or a local *minimum* (if velocity is negative).

Also check speed at the **start** and **end** of the timeframe, as these may also exceed the speeds at the turning points.

Integration is the reverse process to differentiation:

- **integrate acceleration with respect to time to find velocity**
- **integrate velocity with respect to time to find displacement**

In summary:

Differentiate ↓		**displacement**	$= s = \int v\,dt$	↑ **Integrate**
	$\frac{ds}{dt} =$	**velocity**	$= v = \int a\,dt$	
	$\frac{dv}{dt} = \frac{d^2s}{dt^2} =$	**acceleration**	$= a$	