A2 Pure – Chapter 1 – Algebraic Methods

Proof

The AS and A2 proof topics will be taught in your Statistics lesson, and separate notes provided.

Algebraic Fractions

To multiply fractions, cancel any common factors then multiply the numerators and multiply the denominators.

To divide fractions, multiply the first fraction by the reciprocal of the second fraction. Don't try to cancel out common factors until after you have rewritten the division as a multiplication.

To add or subtract two fractions, find a common denominator. This is often the product of the initial denominators.

Partial Fractions

A single fraction with two distinct linear factors in the denominator can be split into the sum of two separate fractions with linear denominators:

$$\frac{px + q}{(ax + b)(cx + d)} \equiv \frac{A}{ax + b} + \frac{B}{cx + d}$$

This is called splitting it into **partial fractions**.

The numerators for the partial fractions can be found using two methods:

1. **Equating coefficients**
   - Multiply through by both linear factors to cancel out the fractions.
   - Expand **all brackets** and collect like terms.
   - The coefficients of each power of x need to be equal on both sides.
   - Use this fact to set up and solve simultaneous equations in A and B.

   This method always works!

2. **Substitution**
   - Multiply through by both linear factors to cancel out the fractions.
   - Substitute the value of x which makes one of the linear brackets equal zero.
   - Repeat for the other linear bracket.
   - These will either give values for A and B, or at worst simultaneous equations with which to find them.

   This method doesn't always work, but can be a useful shortcut when it does.

Harder problems involving partial fractions are often best solved using a combination of the two methods.

Cubic Denominators

The method of partial fractions still works if you have more than two distinct linear factors in the denominator, for example a cubic with three distinct factors.

Simply set up a partial fraction for each factor:

$$\frac{px^2 + qx + r}{(ax + b)(cx + d)(ex + f)} \equiv \frac{A}{ax + b} + \frac{B}{cx + d} + \frac{C}{ex + f}$$

Repeated Linear Factors

A single fraction with a **repeated** linear factor (shown by a squared bracket) can still be split into partial fractions with a slight adjustment to the method. You need two partial fractions for the repeated factor – one with the linear factor as the denominator, and one with the linear factor squared:

$$\frac{px^2 + qx + r}{(ax + b)(cx + d)^2} \equiv \frac{A}{ax + b} + \frac{B}{cx + d} + \frac{C}{(cx + d)^2}$$

The substitution method on its own will not fully solve this problem, as there are only two values of x to substitute but three constants to find. The most efficient method is to use substitution to get as far as you can, then start to compare coefficients. Alternatively, you can use the coefficient method from the start.

Improper Fractions

You may have noticed in the previous generalised examples that the **order** or **degree** of the numerator was always one less than the denominator. Where the denominator was quadratic, we used a linear numerator. Where the denominator was cubic, we used a quadratic numerator. These were all **proper fractions**.

An algebraic fraction is **improper** if **the numerator has a degree equal to or greater than the denominator**.

An improper fraction must be converted to a mixed fraction before you can use partial fractions:

- Divide the numerator by the denominator using **algebraic division** (AS Pure, Chapter 7).
- The result of the division will be the first part of your answer.
- Write the remainder as a fraction of the denominator.
- This fraction will be proper, and so you can now split it into partial fractions.
- Don't forget to combine the result of your division with the partial fractions to get your final answer.

Further Mathematics

You are expected to be able to use partial fractions in Further Mathematics. Generally the techniques described above are all that is required, but on very rare occasions you may be given a denominator with a quadratic that doesn't factorise, in which case you can use this set-up:

$$\frac{px^2 + qx + r}{(ax + b)(cx^2 + dx + e)} = \frac{A}{ax + b} + \frac{Bx + C}{cx^2 + dx + e}$$

From this, equating coefficients will give you three simultaneous equations in A, B and C to solve.