Reciprocal Trigonometric Functions

Secant (sec), cosecant (cosec) and cotangent (cot) are known as the reciprocal trigonometric functions:

$$\sec x = \frac{1}{\cos x} \qquad \text{cosec}\, x = \frac{1}{\sin x} \qquad \cot x = \frac{1}{\tan x}$$

You can also write $\cot x$ in terms of $\sin x$ and $\cos x$:

$$\tan x = \frac{\sin x}{\cos x} \quad \Rightarrow \quad \cot x = \frac{\cos x}{\sin x}$$

$y = \sec x$

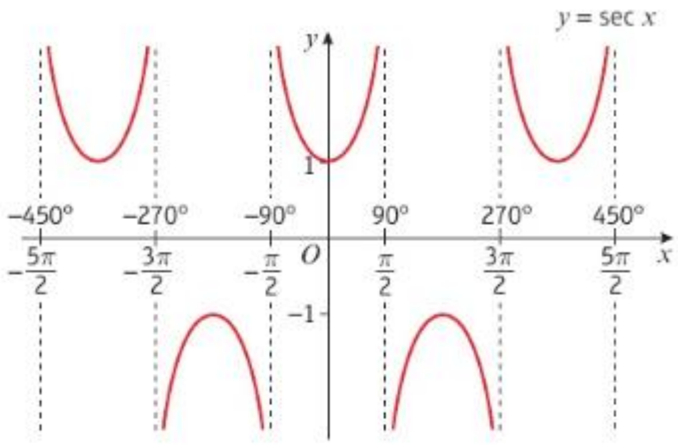
Symmetry in the y-axis, period $360°$ or 2π radians

Domain: $x \in \mathbb{R}, x \neq 90°, 270°, 450° \ldots$

Domain: $x \in \mathbb{R}, x \neq \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$

Domain: $x \in \mathbb{R}, x \neq \frac{(2n+1)\pi}{2}, n \in \mathbb{Z}$

Range: $y \leq -1$ or $y \geq 1$



$y = \text{cosec}\, x$

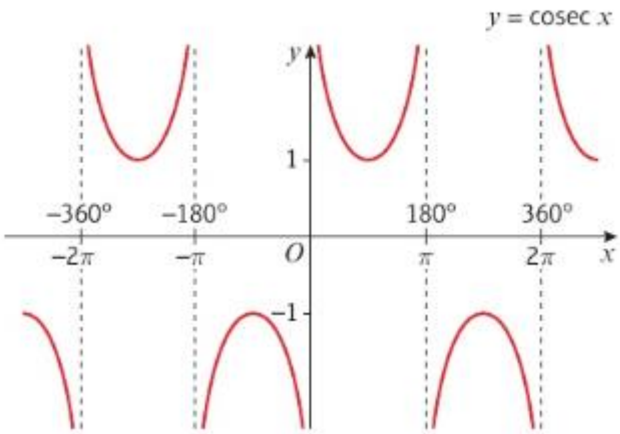
Rotational symmetry, period $360°$ or 2π radians

Domain: $x \in \mathbb{R}, x \neq 0°, 180°, 360° \ldots$

Domain: $x \in \mathbb{R}, x \neq 0, \pi, 2\pi \ldots$

Domain: $x \in \mathbb{R}, x \neq n\pi, n \in \mathbb{Z}$

Range: $y \leq -1$ or $y \geq 1$



$y = \cot x$

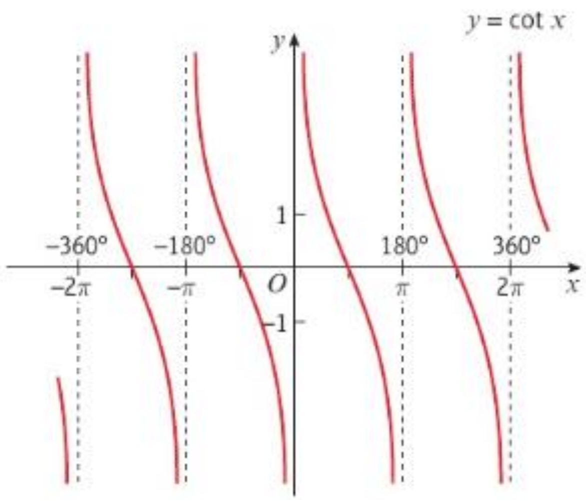
Rotational symmetry, period $180°$ or π radians

Domain: $x \in \mathbb{R}, x \neq 0°, 180°, 360° \ldots$

Domain: $x \in \mathbb{R}, x \neq 0, \pi, 2\pi \ldots$

Domain: $x \in \mathbb{R}, x \neq n\pi, n \in \mathbb{Z}$

Range: $y \in \mathbb{R}$



You can work out the shape of these graphs by considering the respective trigonometric graphs and working out what their reciprocal graphs would look like.

Trigonometric Identities

You already know that: $$\sin^2 x + \cos^2 x \equiv 1$$

This identity can also be written as $\sin^2 x \equiv 1 - \cos^2 x$ and $\cos^2 x \equiv 1 - \sin^2 x$

These two alternative forms allow you to replace $\sin^2 x$ with an expression in $\cos^2 x$ and vice versa. You have used this when solving quadratics in trigonometric functions, and it also has uses in calculus and complex numbers.

There are two equivalent identities that can be derived from $\sin^2 x + \cos^2 x \equiv 1$

Dividing through by $\cos^2 x$ gives

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} \equiv \frac{1}{\cos^2 x} \quad \rightarrow \quad \tan^2 x + 1 \equiv \sec^2 x$$

Dividing through by $\sin^2 x$ gives

$$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} \equiv \frac{1}{\sin^2 x} \quad \rightarrow \quad 1 + \cot^2 x \equiv \operatorname{cosec}^2 x$$

The identity connecting $\tan^2 x$ and $\sec^2 x$ is particularly useful in calculus, especially in Core Pure in Further Maths.

NONE OF THESE IDENTITIES ARE GIVEN IN THE EXAM!

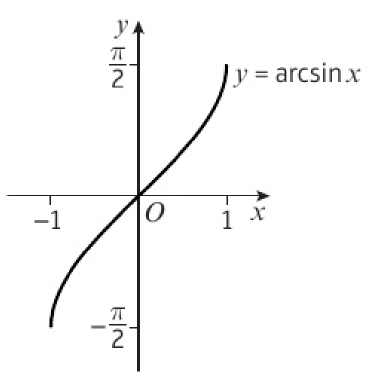
Inverse Trigonometric Functions

The inverse functions of $\sin x$, $\cos x$ and $\tan x$ are $\mathbf{arcsin}\, x$, $\mathbf{arccos}\, x$ and $\mathbf{arctan}\, x$ respectively.

These are strictly defined for a one-to-one section of their trigonometric functions and respective graphs (0 to π for $\arccos x$, $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ for $\arcsin x$ and $\arctan x$), and so give only the **principal value** for each.

As with all **inverse functions**, the graphs are **reflections** of the trigonometric graphs **in the line $y = x$** for the permitted domain and range.

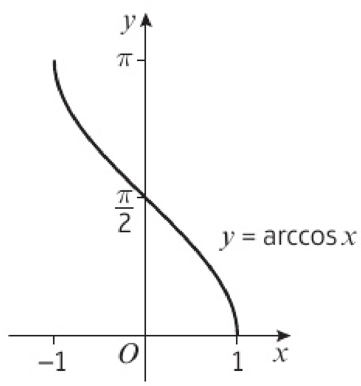
arcsin x



Domain: $-1 \leq x \leq 1$

Range: $-\frac{\pi}{2} \leq \arcsin x \leq \frac{\pi}{2}$

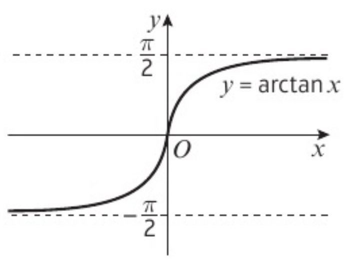
arccos x



Domain: $-1 \leq x \leq 1$

Range: $0 \leq \arccos x \leq \pi$

arctan x



Domain: $x \in \mathbb{R}$ (any real value)

Range: $-\frac{\pi}{2} < \arctan x < \frac{\pi}{2}$