A2 Pure – Chapter 3 – Sequences and Series

Definitions and Notation

A **sequence** is a set of numbers in a given order.

The letter u is used to denote a **term** in a sequence, with a subscript denoting its position in the sequence:

$$\boldsymbol{u_1} \ , \ \boldsymbol{u_2} \ , \ \boldsymbol{u_3} \ , \ \dots \ , \ \boldsymbol{u_n} \qquad \text{with general term } \boldsymbol{u_k}$$

Sequences are defined algebraically in two ways:

Deductively – position-to-term: $u_k = 3k + 1$ (this is the same as "nth term")

Inductively – term-to-term: $u_{k+1} = u_k + 3$ and $u_1 = 4$ ("start at 4 and add 3 each time")

Both the above examples describe the sequence $4 , 7 , 10 , 13 , \dots$

A sequence defined inductively always has a **recurrence relation** of the form $\boldsymbol{u_{n+1} = f(u_n)}$

A **series** is the sum of the terms in a sequence.

The Greek capital letter "**Sigma**" is used to signify a sum. You write it as Σ, with limits on the bottom and top to show which terms you are summing. For example:

$$\sum_{r=1}^{20} 5r = 5 \times 1 + 5 \times 2 + 5 \times 3 + \dots + 5 \times 20$$

Types of Sequence

Arithmetic: Terms increase (or decrease) by the addition of a fixed amount.

$5 , 8 , 11 , 14 , 17$ is arithmetic, with **common difference** 3.

Geometric: Terms are multiplied by a fixed amount to give the next term.

$10 , 20 , 40 , 80 , 160$ is geometric, with **common ratio** 2.

Periodic: A sequence which repeats itself at regular intervals.

$2 , 4 , 6 , 8 , 2 , 4 , 6 , 8 , 2 , 4 , 6 , 8$ is periodic, with **period** 4.

Oscillating: A sequence whose terms lie alternately above and below a middle value.

$6 , 4 , 6 , 4 , 6 , 4$ oscillates with **middle value** 5.

Convergent: A sequence which tends toward a particular value.

$8 , 4 , 2 , 1 , \frac{1}{2} , \frac{1}{4}$ is convergent, converging towards zero

Arithmetic Sequences and Series

In an **arithmetic sequence**, there is a **constant difference** between consecutive terms.

a is used to denote the first term, d is the common difference and n is the number of terms in the sequence.

The formula for the nth term of an arithmetic sequence is $\boldsymbol{u_k = a + (n-1)d}$

The sum of the first n terms of an **arithmetic series** is given by $\boldsymbol{S_n = \frac{n}{2}[2a + (n-1)d]}$

This can also be written, more intuitively, as $\boldsymbol{S_n = \frac{n}{2}(a + l)}$

Geometric Sequences and Series

In a **geometric sequence**, there is a **common ratio** between consecutive terms.

a is used to denote the first term, r is the common ratio and n is the number of terms in the sequence.

The common ratio r can be calculated by $\boldsymbol{\frac{u_{n+1}}{u_n}}$

The formula for the nth term of a geometric sequence is $\boldsymbol{u_n = ar^{n-1}}$

The sum of the first n terms of a **geometric series** is given by $\boldsymbol{S_n = \frac{a(1-r^n)}{1-r}}$, $r \neq 1$ or $\boldsymbol{S_n = \frac{a(r^n-1)}{r-1}}$, $r \neq 1$

A geometric series is **convergent** if and only if $|r| < 1$

The sum to infinity of a geometric series is given by $\boldsymbol{S_n = \frac{a}{1-r}}$