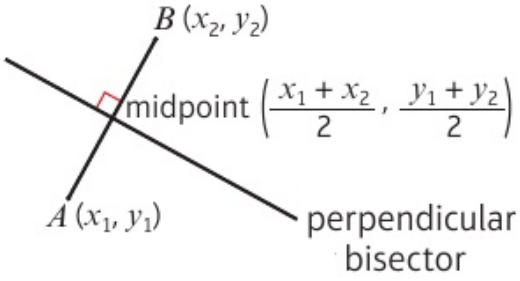
AS Pure – Chapter 6 – Circles

Midpoints and Perpendicular Bisectors

A **line segment** is a finite section of a straight line with two distinct endpoints.

The **midpoint** of a line segment is found by **averaging the x- and y-coordinates of its endpoints**.

The **perpendicular bisector** of a line segment AB is the straight line that is **perpendicular** to AB and **passes through the midpoint** of AB.



To find an equation for the perpendicular bisector of AB, you need the same information as for any straight line

- a point on the line (use the midpoint of AB)
- the gradient (if the gradient of AB is m, this will be $-\frac{1}{m}$)

Equations of Circles

The equation of a circle with centre $O = (0\,,0)$ and radius r is

$$x^2 + y^2 = r^2$$

The equation of a circle with centre $(a\,,b)$ and radius r is

$$\boldsymbol{(x-a)^2 + (y-b)^2 = r^2}$$

The equation of a circle can be given in expanded form and set equal to zero:

$$x^2 + y^2 - 2ax - 2by + a^2 + b^2 - r^2 = 0$$

$$x^2 + y^2 - \alpha x - \beta y + \gamma = 0$$

If you are given the equation in the form, **complete the square** for both the x and y terms and rearrange into the form $(x-a)^2 + (y-b)^2 = r^2$ so that you can read off the centre and radius.

Intersections of Straight Lines and Circles

If you know the equations for a circle and a straight line, you can **solve simultaneously** to find where (if at all) the line touches or intersects the circle.

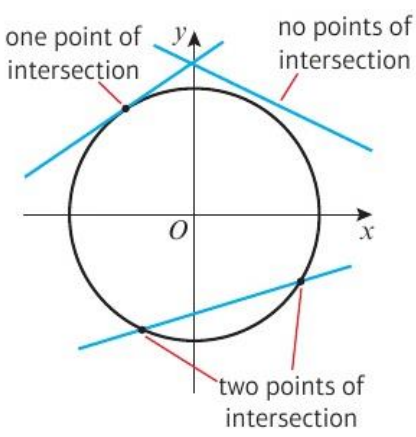
Rearrange the equation of the straight line into the form $y =$ or $x =$ and substitute into the equation of the circle to get a quadratic in one variable.

If this quadratic has no solutions, the line doesn't touch the circle

If this quadratic has two solutions, the line intersects the circle at two points

If this quadratic has one solution, the line touches the circle at a single point – in this case, the line is a **tangent** to the circle

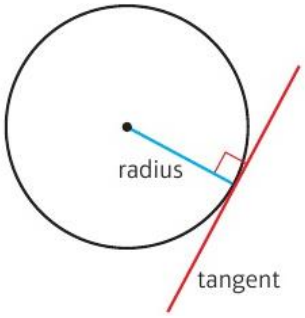
You can use the **discriminant** to check whether the line touches, cuts or misses the circle without having to solve.



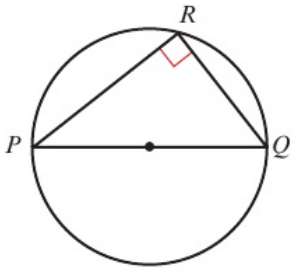
Tangent and Chord Properties

You are expected to be familiar with certain properties of circles related to GCSE circle theorems.

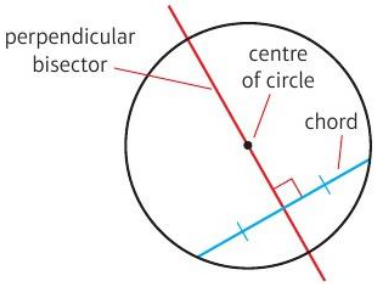
A tangent to a circle is perpendicular to the radius of the circle at the point of intersection:



The angle in a semicircle is always a right angle:



The perpendicular bisector of a chord will pass through the centre of the circle:



We can use this to find the centre of a circle given **any three points** on the circumference:

- Find the equations of the perpendicular bisectors of chords between two different pairs of points
- Find the coordinates where these two bisectors meet – this is the **centre** of the circle
- You can also find the radius by finding the distance from the centre to any point on the circumference