AS Statistics – Chapter 6 – Probability Distributions – Part 2: The Binomial Distribution

When carrying out a number of trials in an experiment or survey, you can define a random variable X to represent the **number of successful trials**.

If the following conditions are met, X can be modelled using a binomial distribution:

- There are a **fixed number of trials, $\boldsymbol{n}$**
- There are **two possible outcomes** (success and failure)
- There is a **fixed probability of success, $\boldsymbol{p}$**
- The trials are **independent** of each other

The notation for this is

$$\boldsymbol{X \sim B(n\,,p)}$$

"The random variable X is modelled with a binomial distribution with parameters n and p"

Probabilities for a Binomial Distribution

If a random variable X has a binomial distribution, then its probability mass function is given by

$$P(X = r) \quad = \quad \binom{n}{r}\, p^r\, (1-p)^{n-r}$$

In this formula,

- $\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$, which represents the number of ways of selecting r successes from n trials
- p^r represents the probability of achieving r successes, each with probability p
- $(1-p)^{n-r}$ represents the probability of $n-r$ successes (the rest of the trials) with probability $1-p$

Cumulative Probabilities from the Binomial Distribution

A **cumulative probability function** for a random variable X tells you the sum of all the individual probabilities **up to and including** the given value of x in the calculation for $P(X \leq r)$ "**probability of $\boldsymbol{r}$ successes or fewer**"

These cumulative probabilities can be worked out on a calculator.

Using a Calculator

To find $P(X = x)$, go to $Menu \rightarrow 7{:}\,Distribution \rightarrow 4{:}\,Binomial\ PD \rightarrow 2{:}\,Variable$ and input values for x, n and p

To find $P(X \leq x)$, go to $Menu \rightarrow 7{:}\,Distribution \rightarrow (down) \rightarrow 1{:}\,Binomial\ CD \rightarrow 2{:}\,Variable$ and input values for x, n and p

Be careful! Other inequalities need a bit more attention!

The table below gives a useful reference guide to the different contexts and their associated inequalities:

Phrase	Means	Calculation
… greater than 5 …	$X > 5$	$1 - P(X \leq 5)$
… no more than 3 …	$X \leq 3$	$P(X \leq 3)$
… at least 7 …	$X \geq 7$	$1 - P(X \leq 6)$
… fewer than 10 …	$X < 10$	$P(X \leq 9)$
… at most 8 …	$X \leq 8$	$P(X \leq 8)$