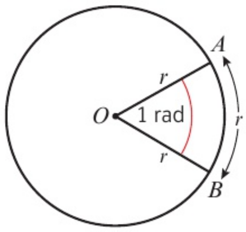
A2 Pure – Chapter 5 – Radians

Definitions

So far, you have only measured angles in **degrees**, where one degree (1°) is defined as $\frac{1}{360}$ of a complete revolution.



You can also measure angles in units called **radians**. One radian is the angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle.

In the diagram, as the arc AB has length r, the angle $\angle AOB$ is exactly 1 radian (1 rad, 1^c)

The circumference of a circle has length $2\pi r$, so you can fit 2π arcs (six and a bit!) of length r into the circumference. This tells us there are 2π radians in a full circle.

Using the same reasoning, one radian is $\frac{1}{2\pi}$ of a full turn, so one radian is $\frac{1}{2\pi} \times 360 = \mathbf{57.295}$ ...°

Conversions

There are 2π radians in a full turn, so

$$\mathbf{360° = 2\pi \ radians} \qquad\qquad \mathbf{180° = \pi \ radians}$$

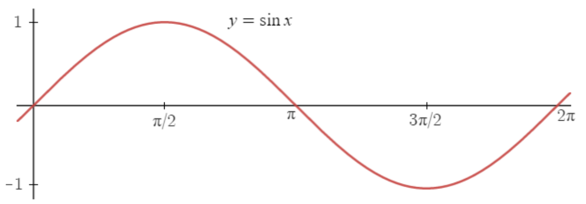
You need to know these! Other key conversions can be memorised or worked out as fractions of $180° = \pi$ radians:

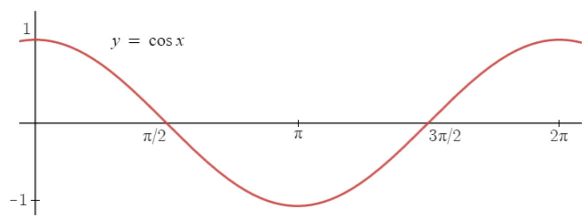
$30° = \frac{\pi}{6}$ radians $\qquad 45° = \frac{\pi}{4}$ radians $\qquad 60° = \frac{\pi}{3}$ radians $\qquad 90° = \frac{\pi}{2}$ radians $\qquad 270° = \frac{3\pi}{2}$ radians

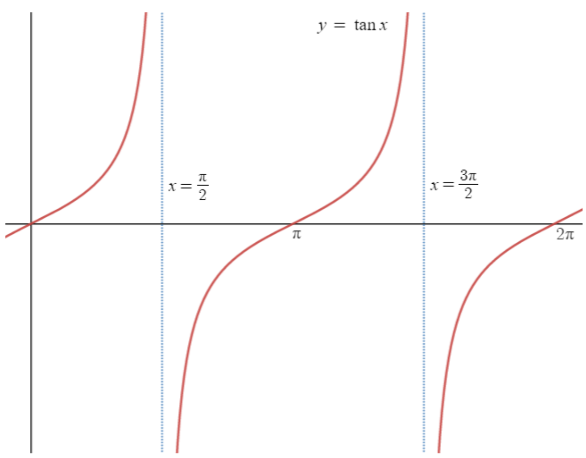
- **To convert from degrees to radians, find the fraction (out of 360) of 2π**
- **To convert from radians to degrees, find the fraction (out of 2π) of 360**

Trigonometric Graphs in Radians

The graphs are exactly the same for angles measured in radians, except the scale on the horizontal axis is changed to match the units. Sine and cosine graphs have period 2π radians, and the tangent graph has period π radians:







Solving Trigonometric Equations in Radians

By setting your calculator to radians, you can solve trigonometric equations in radians just like you would in degrees:

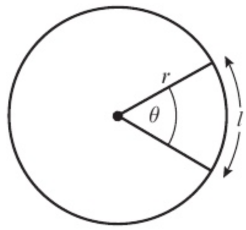
- use the inverse trigonometric functions to get the principal value
- use the symmetry of the graphs to find any other solutions in the range

Careful! Remember you are working in radians, so you will need to add or subtract from $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$ and 2π rather than 90, 180, 270 and 360 when finding additional values.

Arcs, Sectors and Segments

Calculating arc lengths and areas of sectors and segments can be done in exactly the same way as you did at GCSE: use the angle to find the correct fraction of the circumference or area of the full circle. The only difference here is that the angles are fractions of 2π rather than 360.

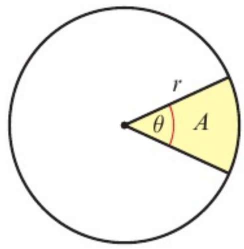
Arc Length: Work out the fraction of the **circumference ($2\pi r$) of the full circle:**



$$\textbf{Arc length} \quad l = \frac{\theta}{2\pi} \times 2\pi r$$

In radians, this can be simplified to $\boldsymbol{l = r\theta}$

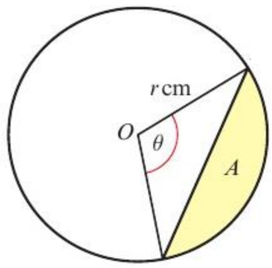
Area of a sector: Work out the fraction of the **area (πr^2) of the full circle:**



$$\textbf{Area of sector} \quad A = \frac{\theta}{2\pi} \times \pi r^2$$

In radians, this can be simplified to $\boldsymbol{A = \frac{1}{2} r^2 \theta}$

Area of a segment: Subtract the area of the triangle from the area of the sector.



$$\textbf{Area of segment} \quad A = \frac{\theta}{2\pi} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$$

The area of the triangle is calculated using the sine area rule $A = \frac{1}{2} ab \sin C$

In radians, this can be simplified to $\boldsymbol{A = \frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin\theta = \frac{1}{2} r^2 (\theta - \sin\theta)}$

Notation

Technically, drawing two radius lines from the centre of a circle creates two sectors, one with an acute or obtuse angle at the centre, and the other with a reflex angle at the centre.

The smaller sector is the **minor sector**, with the **minor arc**.

The larger sector is the **major sector**, with the **major arc**.

Small Angle Approximations

You can use radians to find **approximations** for the values of $\sin\theta$, $\cos\theta$ and $\tan\theta$.

When θ is small (close to zero) **and measured in radians,**

$$\sin\theta \approx \theta$$

$$\cos\theta \approx 1 - \frac{\theta^2}{2}$$

$$\tan\theta \approx \theta$$

These results are given in the formula book.