AS Pure – Chapter 3 – Equations and Inequalities

Linear Simultaneous Equations

Given two linear equations to solve simultaneously, you have two options:

- Rearrange both equations into the form $ax + by = c$ and use an **elimination** method.
- Rearrange one equation into the form $y =$ … or $x =$ … and **substitute** into the other.

Linear and Quadratic Simultaneous Equations

For simultaneous equations with one linear and one quadratic, **substitute the linear into the quadratic**.

These can have up to two pairs of solutions. After substituting, you will have a quadratic in one variable for which you can check the **determinant** to see how many solutions there are. Remember:

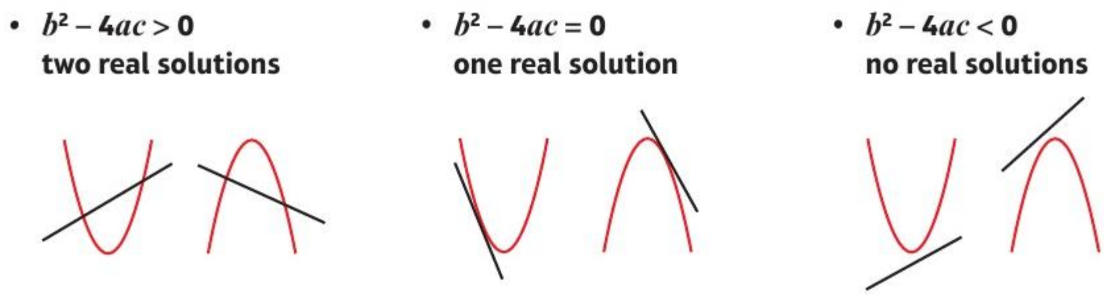
- If $b^2 - 4ac > 0$, then the quadratic function has two real roots.
- If $b^2 - 4ac = 0$, then the quadratic function has one repeated real root.
- If $b^2 - 4ac < 0$, then the quadratic function has no real roots.

If one of the equations has an unknown constant, you can use the discriminant to find a range of values of the constant for which the equations have one, two or zero solutions.

Simultaneous Equations and Graphs

The solutions to a pair of simultaneous equations represent the **points of intersection** of their graphs.

The graphs of a linear and a quadratic equation can intersect once, twice or not at all. As with the solutions to the simultaneous equations, this depends on the discriminant of the quadratic produced by the two equations:



Linear Inequalities

The solution of an inequality is the set of all real numbers for which the inequality is true.

The method for solving a linear inequality is exactly the same as solving a linear equation.

Just remember that when you multiply or divide through by a negative, the inequality sign is reversed.

Set Notation

Solutions to inequalities can also be given in **set notation**. Here are a few examples:

Inequality	Set Notation	Inequality	Set Notation
$x > 7$	$\{x : x > 7\}$	$-2 < x \leq 5$	$\{x : -2 < x \leq 5\}$
$x \leq -4$	$\{x : x \leq -4\}$	$x < 3$ or $x \geq 7$	$\{x : x < 3\} \cup \{x : x \geq 7\}$

Quadratic Inequalities

To solve a quadratic inequality:

- Rearrange so that the right-hand side of the inequality is 0
- Solve the corresponding quadratic equation to find the **critical values**
- Sketch the graph of the quadratic function (∪ or ∩ with roots is all that's needed here)
- Use your sketch to find the required values

When $f(x) > 0$, the points on the graph $y = f(x)$ must have positive y-coordinates. The inequality is therefore represented by the parts of the graph above the x-axis.

Similarly, $f(x) < 0$ is the part of the graph with negative y-coordinates, below the x-axis.

This idea can be applied to quadratics. Generally, for a quadratic equation $ax^2 + bx + c = 0$ with positive a,

- $\boldsymbol{ax^2 + bx + c > 0}$ is solved by the two parts of the graph **outside the roots**
- $\boldsymbol{ax^2 + bx + c < 0}$ is solved by the central part of the graph **between the roots**

Finding the values of x where $ax^2 + bx + c = 0$ gives the roots so you can identify these regions.

If a is negative, $ax^2 + bx + c > 0$ is between the roots and $ax^2 + bx + c < 0$ is outside, as the graph is 'upside down'. This can get confusing, which is why you should always draw a sketch!

Inequalities on Graphs

You may be asked to interpret graphically the solutions to inequalities by considering the graphs of related functions:

- The values of x for which the curve $y = f(x)$ is **below** $y = g(x)$ satisfy the inequality $f(x) < g(x)$
- The values of x for which the curve $y = f(x)$ is **above** $y = g(x)$ satisfy the inequality $f(x) > g(x)$

You can also shade areas on graphs to identify regions which entirely satisfy inequalities:

- $\boldsymbol{y < f(x)}$ represents the area on the coordinate grid **below** the curve $y = f(x)$
- $\boldsymbol{y > f(x)}$ represents the area on the coordinate grid **above** the curve $y = f(x)$

Exclusive inequalities:

If $y > f(x)$ or $y < f(x)$, the curve $y = f(x)$ **is not included** and is represented by a **dotted** line.

Inclusive inequalities:

If $y \geq f(x)$ or $y \leq f(x)$, the curve $y = f(x)$ **is included** and is represented by a **solid** line.