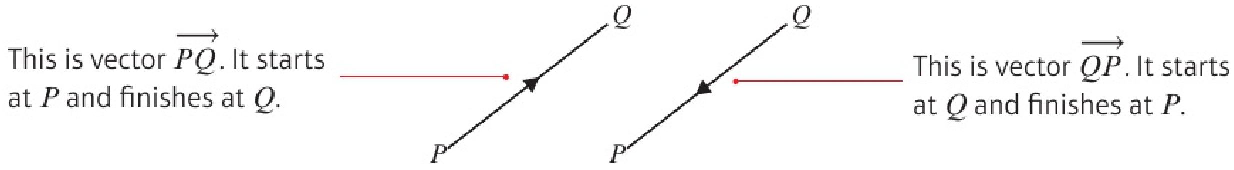
AS Pure – Chapter 11 – Vectors

A **vector** has both **magnitude** and **direction**, and can be represented using a **directed line segment**.



If $\overrightarrow{PQ} = \overrightarrow{RS}$, then the line segments PQ and RS are equal in length and parallel.

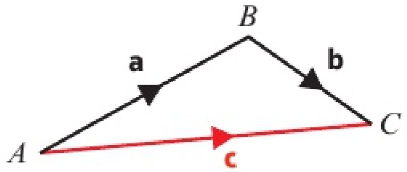
$\overrightarrow{AB} = -\overrightarrow{BA}$ as the line segment AB is equal in length, parallel and in the opposite direction to BA

Addition and Subtraction

You can add two vectors together using the **triangle law** for vector addition:

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

If $\overrightarrow{AB} = \boldsymbol{a}$, $\overrightarrow{BC} = \boldsymbol{b}$ and $\overrightarrow{AC} = \boldsymbol{c}$, then $\quad \boldsymbol{a} + \boldsymbol{b} = \boldsymbol{c}$



Note the use of **bold type** to denote vector quantities. When writing vectors by hand, you can use $\underline{a}$ instead of $\boldsymbol{a}$

Subtracting a vector is equivalent to adding the corresponding negative vector: $\boldsymbol{a} - \boldsymbol{b} = \boldsymbol{a} + (-\boldsymbol{b})$

Adding the vectors $\overrightarrow{AB}$ and $\overrightarrow{BA}$ gives the zero vector: $\overrightarrow{AB} + \overrightarrow{BA} = \mathbf{0}$

Scaling Vectors

Any vector parallel to the vector $\boldsymbol{a}$ may be written as $\lambda\boldsymbol{a}$, where λ is a non-zero scalar.

λ is the scale factor by which $\boldsymbol{a}$ is stretched.

To multiply a column vector by a scalar, multiply each element by the scalar:

$$\lambda\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \lambda a \\ \lambda b \end{pmatrix}$$

To add two column vectors, add the x- and y- components separately: $\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a+c \\ b+d \end{pmatrix}$

The magnitude (length) of a vector $\boldsymbol{a} = \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{pmatrix}$ is calculated using Pythagoras: $|\boldsymbol{a}| = \sqrt{\boldsymbol{x}^2 + \boldsymbol{y}^2}$

A unit vector is a vector of length 1, so a unit vector in the direction of $\boldsymbol{a}$ is: $\hat{\boldsymbol{a}} = \dfrac{\boldsymbol{a}}{|\boldsymbol{a}|} = \dfrac{\mathbf{1}}{|\boldsymbol{a}|}\boldsymbol{a}$

$\boldsymbol{i} - \boldsymbol{j}$ notation

The unit vectors parallel to the x- and y- axes are usually denoted by $\boldsymbol{i}$ and $\boldsymbol{j}$ respectively, so $\boldsymbol{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\boldsymbol{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

For any two-dimensional vector $\begin{pmatrix} x \\ y \end{pmatrix} = x\boldsymbol{i} + y\boldsymbol{j}$

<u>Position Vectors</u>

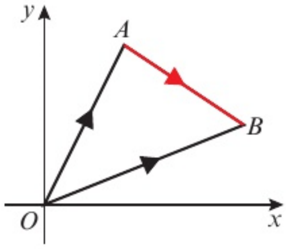
Position vectors are vectors giving the position of a point, relative to a fixed origin O.

The position vector of a point A is the vector $\overrightarrow{OA}$. If $\overrightarrow{OA} = x\boldsymbol{i} + y\boldsymbol{j}$, then the position vector of A is $\binom{x}{y}$

In general, a point P with coordinates $(p\ , q)$ has a position vector $\overrightarrow{OP} = pi + qj = \binom{p}{q}$

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = \overrightarrow{OB} - \overrightarrow{OA},$$

where $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are the position vectors A and B respectively.



If two vectors are parallel and pass through the same point, then they must lie on the same straight line

If you can show that the vector $\overrightarrow{AB}$ is parallel to the vector $\overrightarrow{AC}$ then points A, B and C must lie on the same line as both vectors pass through the point A in the same direction (or the *opposite* direction).

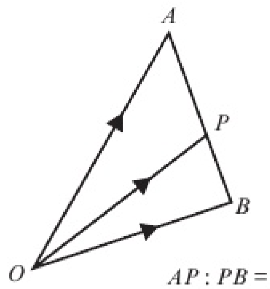
We would call vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ **colinear**.

You need to be able to solve geometric problems and to find the position vector of a point that divides a line segment in a given ratio.

If a point P divides the line segment AB in the ratio $\lambda : \mu$, then

$$\overrightarrow{AP} = \frac{\lambda}{\lambda+\mu}\overrightarrow{AB} \quad \text{and} \quad \overrightarrow{PB} = \frac{\mu}{\lambda+\mu}\overrightarrow{AB}$$

and $$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = \overrightarrow{OA} + \frac{\lambda}{\lambda+\mu}\overrightarrow{AB}$$



$AP : PB = \lambda : \mu$