AS Pure – Chapter 4 – Graphs and Transformations

Roots

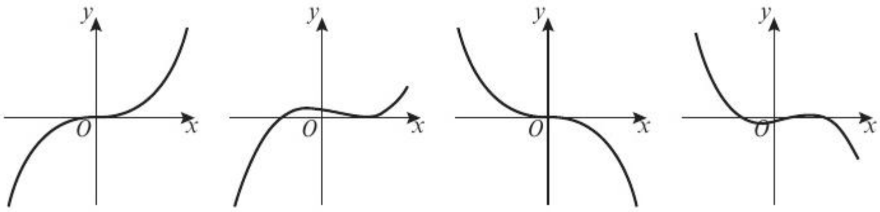
If p is a **root** of a function $f(x)$, then the graph of $y = f(x)$ touches or crosses the x-axis at the point $(p, 0)$

You already know that a **quadratic** can have one, two or zero roots, depending on whether its graph touches, crosses or is entirely above or below the x-axis. You can use the **discriminant** to check how many roots a quadratic has.

Cubic Graphs

A **cubic function** has the form $f(x) = ax^3 + bx^2 + cx + d$, where a, b, c, d are real numbers and $a \neq 0$

The graph of a cubic function can take a few similar but distinct forms, depending on the number of roots and stationary points, which in turn depend on the values of the constants.



For these two functions a is positive. For these two functions a is negative.

A cubic graph always starts and finishes in opposite quadrants, so it must cross the x-axis at least once. As such, a cubic has **at least one** and **up to three real roots**. It can only have exactly two real roots if one is repeated (so the curve touches the axis).

When sketching a cubic, bear in mind the following:

- The sign of the x^3 term determines which way up the graph needs to be sketched.
- Find the roots by factorising. You may need trial and error to find your first root.
- For a repeated root, the curve will touch rather than cross the x-axis.
- Don't forget to mark the y-intercept.
- Are you asked to find stationary points? If so use differentiation to find $f'(x)$, set equal to zero and solve
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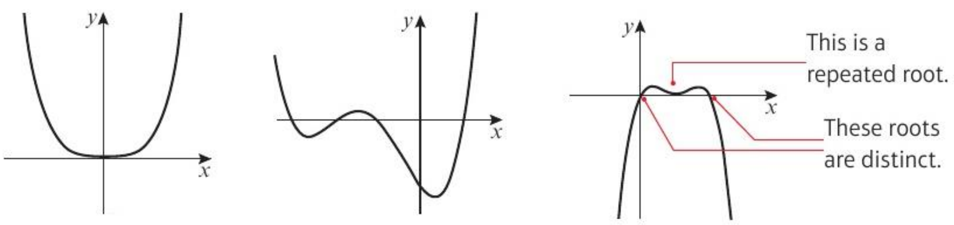
Quartic Graphs

A **quartic function** has the form $f(x) = ax^4 + bx^3 + cx^2 + dx + e$, where a, b, c, d, e are real numbers and $a \neq 0$

The graph of a quartic function can take also take several distinct forms, depending on the number of roots and stationary points, but the general form is roughly a curved W-shape, or M-shape if the x^4 term is negative



For these two functions a is positive. For this function a is negative.

A cubic graph always starts and finishes the same side of the x-axis (above or below) so it doesn't have to cross. As such, a quartic has anywhere from **zero to four real roots**.

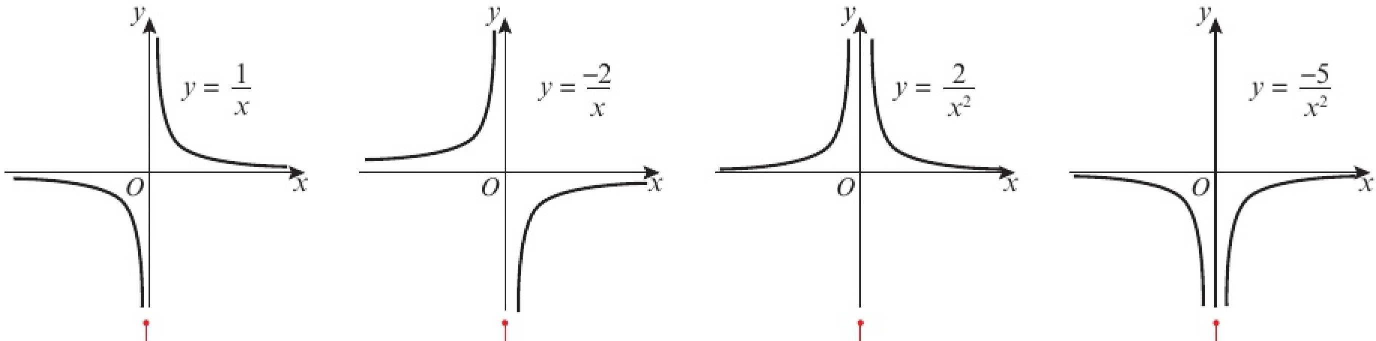
Sketch the curve as you would a quadratic or cubic, by marking the roots and fitting the general shape around them.

Reciprocal Graphs

You can sketch graphs of **reciprocal functions** by considering their signs and their asymptotes.

An **asymptote** is a line which the curve approaches but never reaches.

The graphs of $y = \frac{k}{x}$ and $y = \frac{k}{x^2}$, where k is a real constant, have asymptotes at $x = 0$ and $y = 0$



$y = \frac{k}{x}$ with $k > 0$. $y = \frac{k}{x}$ with $k < 0$. $y = \frac{k}{x^2}$ with $k > 0$. $y = \frac{k}{x^2}$ with $k < 0$.

Reciprocal graphs for x have a section either side of the x-axis, but for x^2 both sections are either above or below.

Points of Intersection

By sketching curves of two functions on the same axes, you can see how many points of intersection there are.

The x-coordinates at the points of intersection of the curves $y = f(x)$ and $y = g(x)$ are the solutions to the equation $f(x) = g(x)$. The x- *and* y-coordinates of the points of intersection are the solutions to the *simultaneous* equations $y = f(x)$ and $y = g(x)$.

Make sure you understand the distinction between these two scenarios!

Transformations

You are expected to be able to transform graphs using the same basic transformations seen at GCSE.

The following are transformations of the graph $y = f(x)$

Horizontal (x-values are transformed **before** the function is applied)**:**

$y = f(x - a) \rightarrow$ translation $\binom{a}{0}$ $\qquad$ $y = f(ax) \rightarrow$ stretch, x-direction, scale factor $\frac{1}{a}$

Vertical (the function is applied to the x-values, **then** transformed)**:**

$y = f(x) + a \rightarrow$ translation $\binom{0}{a}$ $\qquad$ $y = af(x) \rightarrow$ stretch, y-direction, scale factor a

Reflections:

$y = f(-x) \rightarrow$ horizontal reflection (in y-axis) $\qquad$ $y = -f(x) \rightarrow$ vertical reflection (in x-axis)

Note that when a function is transformed, any asymptotes are also transformed. The transformation can also affect which values of x are excluded from the domain of the graph. For example, for $y = \frac{1}{x}$, we know that $x \neq 0$, but for the transformed graph $y = \frac{1}{x-2}$, we have $x \neq 2$ instead.