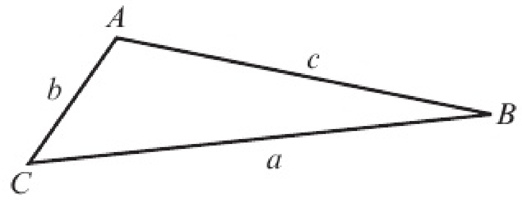
AS Pure – Chapter 9 – Trigonometric Ratios

The sine and cosine rules can be used to find missing sides and angles for any triangle.

The rules are given based on a triangle with sides a, b, c with corresponding opposite angles A, B, C



The Cosine Rule

To find the missing side in a triangle when you know the other two sides and the angle between them, use:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

To find a missing angle given all three sides, rearrange the cosine rule and use:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Sine Rule

To find a missing side when you know the opposite angle and another side-angle pair, use:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

To find a missing angle when you know the opposite side and another side-angle pair, use:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Note that your calculator will only give values between $0°$ and $90°$ for the angle.

There may be a second possible solution between $90°$ and $180°$. To find this, subtract the first angle from 180.

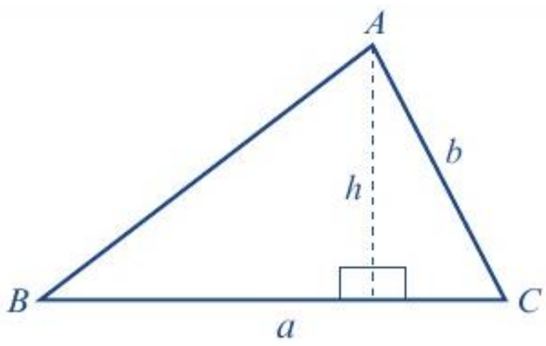
This works because $\sin\theta = \sin(180 - \theta)$, as you can see from the graph of the sine function.

Area of a Triangle

You can find the area of any triangle if you know two sides and the angle between them:

$$Area = \frac{1}{2}\, a\, b \sin C$$

You may be asked to derive this formula:



Area of a triangle = ½ x base x perpendicular height

In this case, $\boldsymbol{Area = \frac{1}{2}ah}$

Using SOHCAHTOA, we can see that $\sin C = \frac{h}{b}$, so $h = b\sin C$

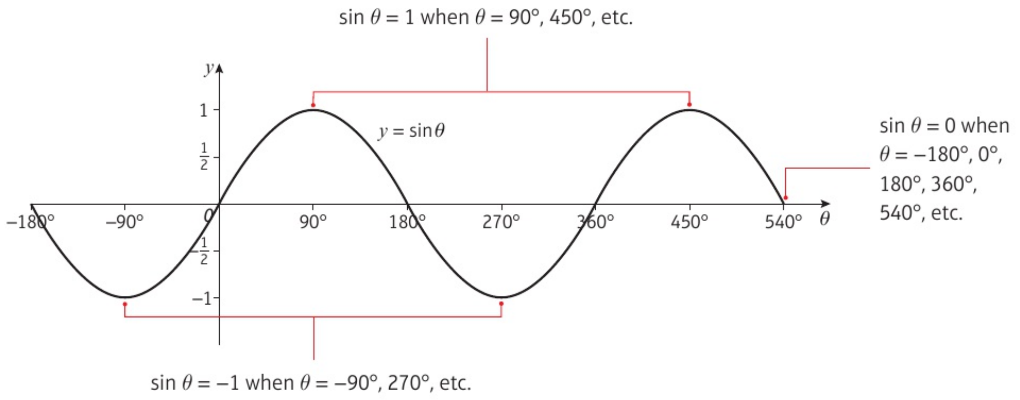
Substituting this expression for h into the formula for the area,

$\boldsymbol{Area = \frac{1}{2}\, a\, b \sin C}$

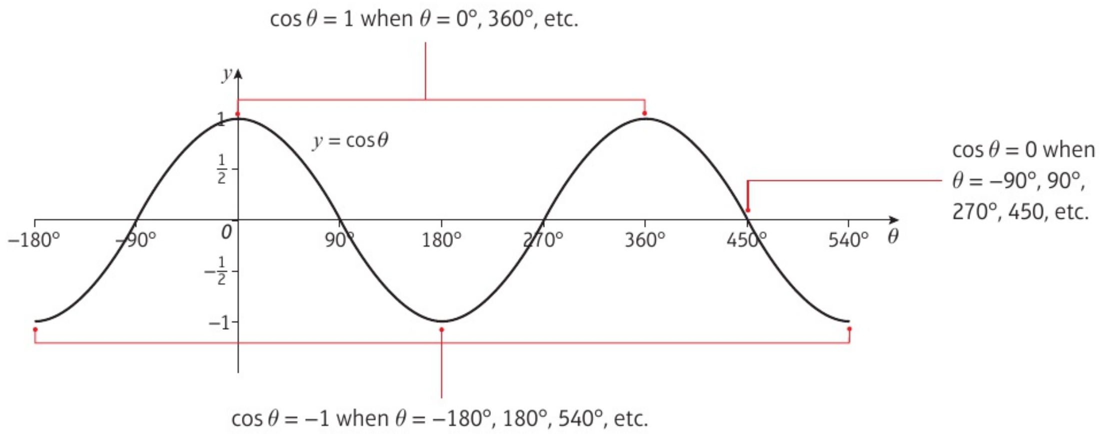
Trigonometric Graphs

The graphs of sine, cosine and tangent are **periodic**, meaning they repeat themselves after a fixed interval.

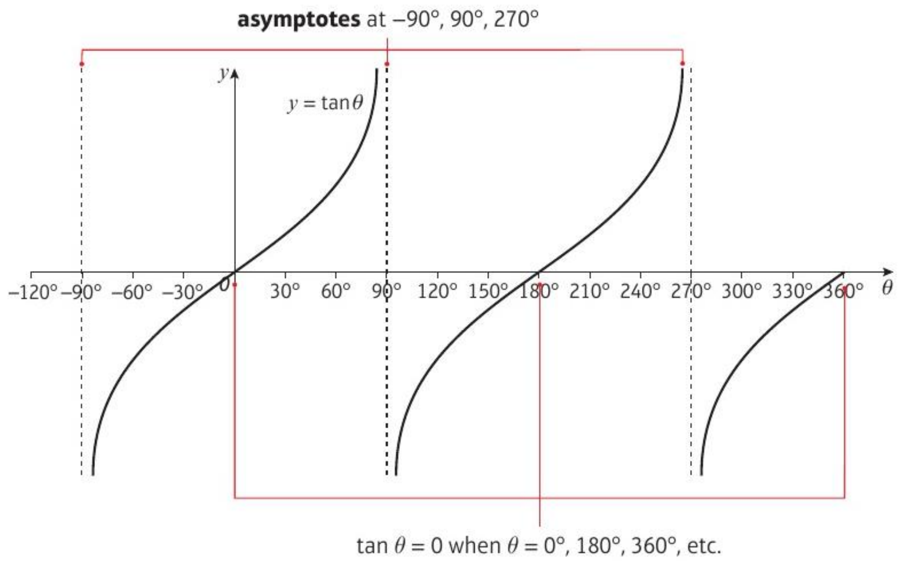
The graph $\boldsymbol{y = \sin\theta}$ repeats every 360°:



The graph $\boldsymbol{y = \cos\theta}$ repeats every 360°:



The graph $y = \tan\theta$ repeats every 180°:



You are expected to be able to transform trigonometric graphs using the same basic transformations seen at GCSE:

Horizontal: $f(x-a) \to$ translation $\begin{pmatrix} a \\ 0 \end{pmatrix}$ $\quad f(ax) \to$ stretch, x-direction, scale factor $\frac{1}{a}$

Vertical: $f(x)+a \to$ translation $\begin{pmatrix} 0 \\ a \end{pmatrix}$ $\quad af(x) \to$ stretch, y-direction, scale factor a

Others: $-f(x) \to$ vertical reflection (in x-axis) $\quad f(-x) \to$ horizontal reflection (in y-axis)