AS Pure – Chapter 7 (except Proof) – Algebraic Division and the Factor Theorem

Algebraic Fractions

You can simplify algebraic fractions using division.

Where possible, you can also **factorise** the numerator and denominator and then cancel common factors.

Dividing Polynomials

A **polynomial** is a finite expression with positive whole number indices (including constants)

Examples include:	$2x + 4$	$4xy^2 + 3x - 9$	8
These are not polynomials:	$\sqrt{x}$	$6x^{-2}$	$\frac{4}{x}$

You can use long division to divide a polynomial in powers of x by a linear binomial $(x \pm p)$, where p is a constant.

Make sure the polynomial is written in **descending** powers of x, and leave **placeholders** for missing powers.

If there is no remainder, you can use the result to write the polynomial as a **product of two factors**.

With practice, you may learn to divide by **inspection**, although this method only works if there is no remainder.

The Factor Theorem

The factor theorem is a quick way of finding simple linear factors of a polynomial.

The factor theorem states that if $f(x)$ is a polynomial, then:

- **If $f(a) = 0$, then $(x - a)$ is a factor of $f(x)$**
- **If $(x - a)$ is a factor of $f(x)$, then $f(a) = 0$**

These statements don't necessarily imply each other. The proof that both are true is beyond the scope of the course.

You can use the factor theorem to factorise a cubic function, $f(x)$, as follows:

1. Substitute values of x into the function until you find a value a such that $f(a) = 0$
2. Divide the function by the factor $(x - a)$. The remainder should be 0, confirming that $(x - a)$ is a factor
3. Write $f(x) = (x - a)(Ax^2 + Bx + C)$. If $f(x)$ is cubic, the other factor will always be a quadratic.
4. Factorise the quadratic factor, if possible, to write $f(x)$ as a product of three linear factors.