AS Pure – Chapter 2 – Quadratics

A quadratic equation can be written in the form $ax^2 + bx + c = 0$, where a, b and c are constants and $a \neq 0$.

a, b and c are called **coefficients**. a is the coefficient of x^2, b is the coefficient of x and c is the constant coefficient.

Quadratic equations can have one, two or no real solutions, also called **roots**.

If a quadratic only has one solution, we call this a **repeated root**.

There are three methods you need to know for solving quadratic equations:

Factorising

- Write the equation in the form $ax^2 + bx + c = 0$
- Factorise the left-hand side
- Set each factor equal to zero and solve to find value(s) for x

Quadratic Formula

- Write the equation in the form $ax^2 + bx + c = 0$
- Substitute a, b and c into the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Completing the Square

- Write the equation in the form $ax^2 + bx + c = 0$
- Rearrange the left-hand side into the form $p(x + q)^2 + r = 0$
- Solve by subtracting r, dividing by p, square rooting and subtracting q
- Don't forget the square root can be positive or negative!

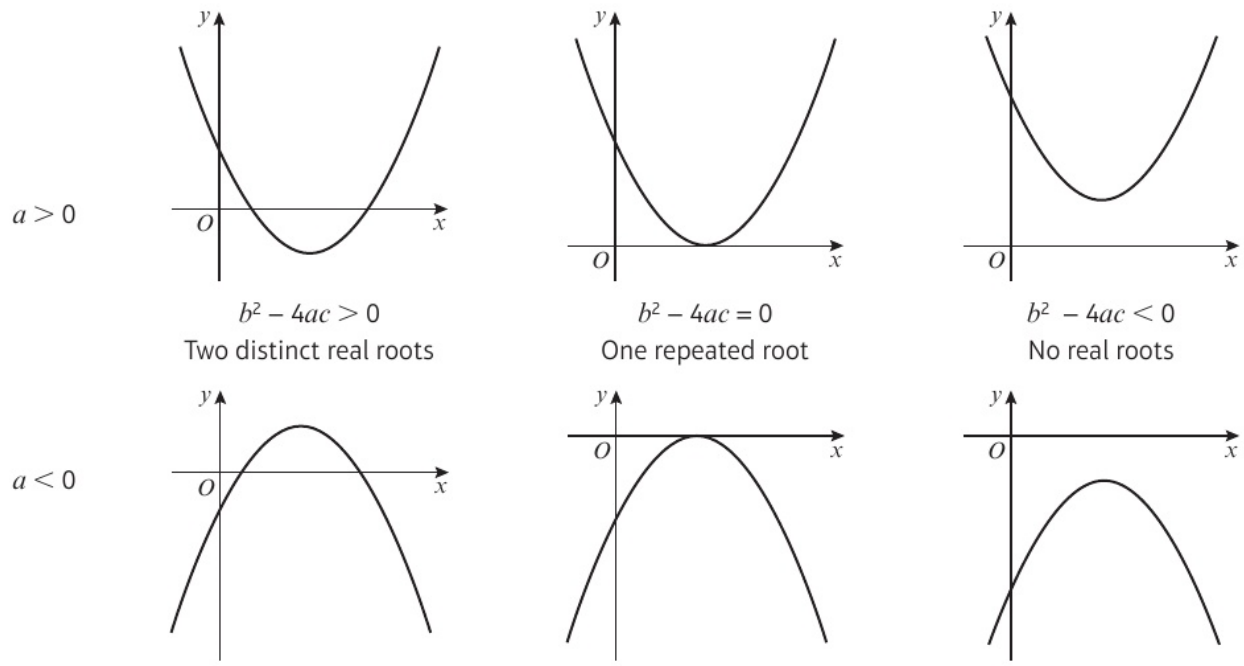
The Discriminant

For a quadratic function $f(x) = ax^2 + bx + c$, the expression $b^2 - 4ac$ is called the **discriminant**.

The value of the discriminant tells us how many real roots $f(x)$ has:

- If $b^2 - 4ac > 0$, then the quadratic function has two real roots.
- If $b^2 - 4ac = 0$, then the quadratic function has one repeated real root.
- If $b^2 - 4ac < 0$, then the quadratic function has no real roots.

The diagrams below show how the value of the discriminant relates to the number of real roots a quadratic has:



Quadratic Graphs

If the coefficient of x^2 is positive, the graph has a ∪ shape. If it is negative, the graph is "upside-down" (∩-shaped)

When sketching a quadratic graph, always label the **points of intersection** with the x-axis and y-axis.

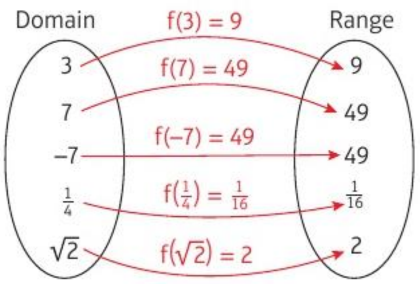
You may also be asked to find the **turning point**, which always has an x-coordinate midway between the roots.

You can also find the turning point by completing the square: if $f(x) = a(x+p)^2 + q$, the turning point is $(-p\,,q)$

Later in the course, you will see that the turning point can also be found using a technique called **differentiation**.

Functions

A function is a mathematical relationship that maps each input to a single output.



The set of possible inputs for a function is called the domain.

The set of possible outputs for a function is called the range.

The diagram to the right shows several mappings in the function $f(x) = x^2$

The roots of a function $f(x)$ are the values of x for which $f(x) = 0$