A2 Pure – Chapter 8 – Parametric Equations

A curve can be defined using parametric equations $\boldsymbol{x = p(t)}$ and $\boldsymbol{y = q(t)}$

Each value of the parameter $\boldsymbol{t}$ defines a point on the curve with Cartesian coordinates $(\,\boldsymbol{p(t)}\,, \boldsymbol{q(t)}\,)$

You can use parametric equations to model a variety of real-world situations. In fact, you've already done so!

In Mechanics, you modelled vertical and horizontal displacement under gravity using the equation $\boldsymbol{s = ut + \frac{1}{2}at^2}$:

$x = ut$ (acceleration $a_x = 0$)

$y = y_0 + ut - 4.9t^2$ (acceleration $a_y = -9.8$)

In this case, the horizontal and vertical positions are defined in terms of the parameter t, which is the time.

Converting to Cartesian equations

You can convert parametric equations to a Cartesian equation by using substitution to eliminate the parameter.

Unless specified, you don't necessarily have to give these equations in the form $y = \ldots$

If the parametric equations are given in terms of trigonometric functions, you may be able to use trigonometric identities to simplify the Cartesian equations.

Domain and Range

Remember that the domain of a Cartesian function $y = f(x)$ is all possible x-values, and the range is the y-values.

For parametric equations $\boldsymbol{x = p(t)}$ and $\boldsymbol{y = q(t)}$ with Cartesian equation $\boldsymbol{y = f(x)}$,

- **The domain of $\boldsymbol{f(x)}$ is the range of $\boldsymbol{p(t)}$** **(as this gives the $\boldsymbol{x}$-coordinates of the curve)**
- **The range of $\boldsymbol{f(x)}$ is the range of $\boldsymbol{q(t)}$** **(as this gives the $\boldsymbol{y}$-coordinates of the curve)**

Parametric Differentiation

If x and y are given as functions of a parameter t, then:

$$\boldsymbol{\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}}$$

(Using the chain rule, by rearranging $\frac{dy}{dx} \times \frac{dx}{dt} = \frac{dy}{dt}$)

Parametric Integration

At A-level, most parametrics can be integrated by rearranging into a Cartesian equation and integrating as normal.

If you're struggling to find a Cartesian form you can integrate, you could also use the chain rule to get:

$$\int y\,dx = \int y\,\frac{dx}{dt}\,dt$$

and substitute expressions for y and $\frac{dx}{dt}$ in terms of t. Remember to change your bounds if you change variable!