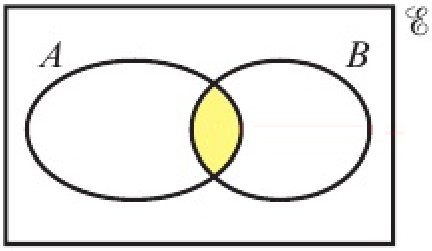
<u>A2 Statistics – Chapter 2 – Conditional Probability</u>

Set Notation and Venn Diagrams

The event "A and B" can be written as $A \cap B$. The symbol ∩ is the symbol for **intersection**.



If A and B are **independent**, then the intersection $P(A \cap B) = P(A) \times P(B)$

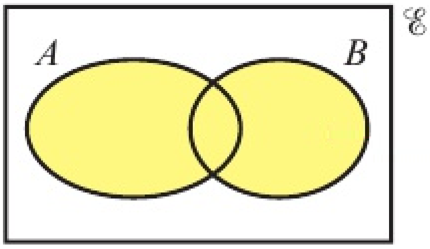
If A and B are **mutually exclusive**, then the intersection is empty.

In this case, $A \cap B = \emptyset$, where ∅ denotes the **empty set**, and $P(A \cap B) = 0$

The symbol ℰ is used to represent the **whole sample space**.

The event "A or B" can be written as $A \cup B$. The symbol ∪ is the symbol for **union**.

Note that the union covers "A or B **or both**"

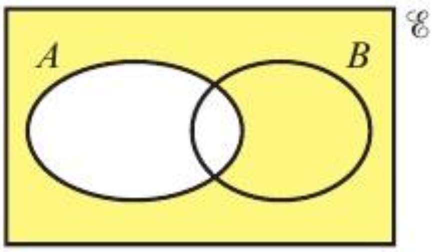


The **union** can be found by adding circles for A and B then subtracting the **intersection**, which was counted twice:

$$\boldsymbol{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Hence, If A and B are **mutually exclusive**, then the union $P(A \cup B) = P(A) + P(B)$

The event "not A" can be written as A'. This is also called the **complement** of A.



Events A and A' are always **mutually exclusive**.

Conditional Probability

The probability that B occurs given that A has already occurred is written as $P(B \mid A)$.

Similarly, the probability that B occurs given that A has not occurred is given as $P(B \mid A')$.

For independent events, $P(B \mid A) = P(B \mid A') = P(B)$

In plain English, this just means that the probability of B occurring is the same regardless of whether A occurs.

This is also true in reverse: $P(A \mid B) = P(A \mid B') = P(A)$ if A and B are independent.

You can solve problems involving conditional probability by considering a **restricted sample space**.

For $P(B \mid A)$, we can consider this probability to be equal to the probability of B occurring in the restricted sample space for which A has already occurred. This is described mathematically as:

$$\boldsymbol{P(B \mid A) = \frac{P(B \cap A)}{P(A)}}$$

This is essentially the probability of both B and A occurring, as a fraction of the probability of A occurring.

This is given in the Formula Book in a rearranged form:

$$P(A \cap B) = P(A) \times P(B \mid A)$$

Checking for Independence

You now have two ways of checking whether two events are independent:

If $\boldsymbol{P(A) \times P(B) = P(A \cap B)}$ - this is usually the easiest check to calculate – use it in the exam!

If $\boldsymbol{P(B|A) = P(B)}$ - use this if you are given or have already calculated $P(B|A)$

Both of these are given in the Formula Book.