A2 Pure – Chapter 4 – Binomial Expansion

You know that if n is a positive integer (*natural number*), you can expand $(a + bx)^n$ using the formula:

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n \qquad (n \in \mathbb{N})$$

and substituting bx for b.

If n is a **fraction** or a **negative number**, you need to use a different version of the binomial expansion:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$$

This expansion is only valid for $|x| < 1$, $n \in \mathbb{R}$

Unless n is a positive integer, none of the coefficients will equal zero and so this version of the binomial expansion will produce an **infinite** number of terms.

Since x is restricted to values between -1 and 1, the terms will tend towards zero as the powers of x increase. This means a finite number of terms can give an approximate value for the expansion.

$(1 + bx)^n$

To expand $(1 + bx)^n$, substitute bx for x and apply the formula as normal.

The expansion of $(1 + bx)^n$ is only valid for $|bx| < 1$, or $|x| < \frac{1}{|b|}$

$(a + bx)^n$

To expand $(a + bx)^n$, take out a factor of a to get $a^n\left(1 + \frac{b}{a}x\right)^n$ and apply the formula as normal.

The expansion of $(a + bx)^n$ is only valid for $|bx| < |a|$, which is equivalent to $|x| < \left|\frac{a}{b}\right|$

Validity for combinations of binomial expansions

If two or more binomial expansions are combined into a single function, the range of values of x for which the function is valid is taken from the binomial with the strictest limits. For example:

$(1 + 3x)^n$ is valid for $|x| < \frac{1}{3}$ $(2 - 5x)^n$ is valid for $|x| < \frac{2}{5}$ so $(1 + 3x)^n(2 - 5x)^n$ is valid for $|x| < \frac{1}{3}$

Multiplying two binomial expansions

When asked to expand a product of two different binomials to get a result up to, say, the term in x^3, expand each binomial separately up to the required term and then multiply the two using a grid. This makes it straightforward to evaluate and collect just the terms that you need, and reduces the risk of errors when multiplying out large brackets.