Y10 Maths Knowledge Organiser Higher Tier: Algebraic Manipulation

What must I be able to do?

New content:

- ☐ Expand a double bracket (two binomials) to give a quadratic expression
  - ➢ Mathswatch 143b (GCSE)
- ☐ Expand more than two binomials
  - ➢ Mathswatch 178 (GCSE)
- ☐ Factorise a quadratic expression into two linear brackets
  - ➢ Mathswatch 157 (GCSE) and 192 (GCSE)
- ☐ Change the subject of a formula where the subject appears more than once
  - ➢ Mathswatch 190 (GCSE)

Key vocabulary

Binomial	An algebraic expression with just 2 terms e.g. $3x + 4$
Quadratic	An algebraic expression where the highest power is 2 e.g. $x^2 + 3x$
Cubic	An algebraic expression where the highest power is 3 e.g. $x^3 - 6x + 1$

Expanding a linear bracket

Multiply all terms inside the bracket by the term in front of the bracket being careful with any negative numbers

e.g. $4(3a - 6) = 12a - 24$

as $4 \times 3a = 12a$ and $4 \times -6 = -24$

Substitution

Replace letters with their known values and then work out the answer

e.g. Given that $a = 4, b = 5, c = -6$

then $a + b = 4 + 5 = 9$ and

$ac + 2b = 4 \times -6 + 2 \times 5 = -24 + 10 = -14$

BIDMAS!

Remember that 2 terms with no sign between mean that you multiply them so $2b$ means $2 \times b$ and ac means $a \times c$

Identify equations, expressions, formulae and identities

Collection of terms with no equals sign

More than one variable and an equals sign

	Expression	Equation	Formula	Identity
$3x + 4$	✓			
$3x + 4 = 12$		✓		
$P = 4x$			✓	
$3x + 12 \equiv 3(x + 4)$				✓

Has an equals sign and only one unknown. Can be solved.

Factorising linear expressions

Factorising is the opposite of expanding a bracket. Find the largest common factors of all terms and divide by these. The factors are put in front of the bracket.

e.g. $12x + 4 = 4(3x + 1)$

$25y + 15 = 5(5y + 3)$

$18a^2 - 4a = 2a(9a - 2)$

Expanding a double bracket

Method 1 – "smiley face"

Draw loops between each pair and multiply the two values at the end of the loops together

$(2x + 4)(3x + 5)$

$2x \times 3x = 6x^2$

$4 \times 3x = 12x$

$12x + 10x = 22x$

$2x \times 5 = 10x$

$4 \times 5 = 20$

So $6x^2 + 22x + 20$

Method 2 – Separate the brackets

In this method we split the pair of brackets back into single ones

$(2x + 4)(3x + 5)$

$= 2x(3x + 5) + 4(3x + 5)$

$= 6x^2 + 10x + 12x + 20$

$= 6x^2 + 22x + 20$

Method 3 - Grid

Set the expansion out as a multiplication grid

$(2x + 4)(3x + 5)$

	$3x$	$+5$
$2x$	$6x^2$	$10x$
$+4$	$12x$	20

So $6x^2 + 22x + 20$

Expanding 3 brackets

First expand the first two brackets using a normal method to get a quadratic. Then use a grid to multiply the quadratic by the third bracket.

$(3x + 2)(2x - 4)(5x + 7)$

First expand $(3x + 2)(2x - 4)$

	$3x$	$+2$
$2x$	$6x^2$	$4x$
-4	$-12x$	-8

So $6x^2 - 8x - 8$

Now multiply out $(6x^2 - 8x - 8)(5x + 7)$

	$6x^2$	$-8x$	-8
$5x$	$30x^3$	$-40x^2$	$-40x$
$+7$	$42x^2$	$-56x$	-56

Diagonal boxes in the grid have similar terms which can be collected together and simplified for the final answer

So the final answer is $30x^3 + 2x^2 - 96x - 56$

Factorising Quadratics

The general form of a quadratic expression is $\mathbf{ax^2 + bx + c}$ where a, b and c are numbers.

Type 1: $a = 1$

When factorising a full quadratic expression, it goes into 2 brackets. The second terms in the brackets need to multiply to make the "+c" and add to make the "+b"

e.g. $x^2 + 8x + 12$

$6 \times 2 = 12$ and $6 + 2 = 8$

$(x + 6)(x + 2)$

$x^2 - 10x + 24$

$-6 \times -4 = 24$ and $-6 + -4 = -10$

$(x - 6)(x - 4)$

$x^2 - 3x + 28$

$-7 \times 4 = -28$ and $-7 + 4 = -3$

$(x - 7)(x + 4)$

Special cases: 1) No "+c" e.g. $6x^2 + 3x$ This factorises into 1 bracket rather than 2. $6x^2 + 3x = 3x(2x + 1)$

2) No "+b" and c is negative e.g. $x^2 - 25$ This is known as the **difference of two squares** and factorises into two brackets. Both brackets are the same except the sign in the middle $x^2 - 25 = (x + 5)(x - 5)$

Type 2: $a > 1$

This method also works for when $a = 1$ but takes slightly longer than just "spotting" it.

e.g. $6x^2 - 11x - 10$

Step 1 – multiply a and c together then find factors of this number which add to b

$6 \times -10 = -60$. Factors of -60 which add to -11 are -15 and $+4$

Step 2 – Rewrite the b term ($-11x$) using these two factors

$6x^2 - 15x + 4x - 10$

Step 3 – Factorise the first two terms into one bracket

$3x(2x - 5) + 4x - 10$

Step 4 – Factorise the last two terms into one bracket. Tip – it will be the same bracket as used for the first two terms

$3x(2x - 5) + 2(2x - 5)$

Step 5 – This bracket is a factor of both terms so now rewrite as two brackets

$(3x + 2)(2x - 5)$

Changing the subject of a formula

This follows the same rules as when solving equations.

e.g. make u the subject of the formula

$$y = 2u + 3p$$

$-3p$ $-3p$

$$y - 3p = 2u$$

$\div 2$ $\div 2$

$$\frac{y - 3p}{2} = u$$

e.g. make c the subject of the formula

$$m = 5(c - 1)$$

There are 2 options here:

Method 1: expand the bracket first

$$m = 5(c - 1)$$

expand   expand

$$m = 5c - 5$$

$+5$ $+5$

$$m + 5 = 5c$$

$\div 5$ $\div 5$

$$\frac{m + 5}{5} = c$$

Method 2: divide by the coefficient first

$$m = 5(c - 1)$$

$\div 5$ $\div 5$

$$\frac{m}{5} = c - 1$$

$+1$ $+1$

$$\frac{m}{5} + 1 = c$$

Tip – examiners tell schools that method 1 usually has a higher success rate in an exam than method 2 does!

Changing the subject of a formula where the wanted subject appears more than once

If the variable needed as the subject appears in more than one place then the first step is to collect all the terms with that variable on one side of the equals, with all other terms on the other side.

e.g. make p the subject of the formula

$$a(p - 2s) = 3p + 2$$

Here p is in 2 places, so the first step is to get both p terms on the left hand side and anything else on the right hand side

$$ap - 2as = 3p + 2$$

$$ap - 2as - 3p = 2$$

$$ap - 3p = 2 + 2as$$

Now to get a single p, **factorise** the left hand side and take p as the factor

$$p(a - 3) = 2 + 2as$$

Finally, divide both sides by the bracket

$$p = \frac{2+2as}{a-3}$$

e.g. make b the subject of the formula

$$a = \frac{2 - 7b}{b - 5}$$

x (b – 5)

$$a(b - 5) = 2 - 7b$$

expand the bracket

$$ab - 5a = 2 - 7b$$

$+ 7b$

$$ab + 7b - 5a = 2$$

$+ 5a$

$$ab + 7b = 2 + 5a$$

factorise

$$b(a + 7) = 2 + 5a$$

$\div (a - 7)$

$$b = \frac{2 + 5a}{a + 7}$$

GLUE

HERE