Y10 Maths Knowledge Organiser Higher Tier: Sequences

What must I be able to do?	Key vocabulary	
New content: ☐ Know and use geometric sequences ➢ Mathswatch 163 (GCSE) ☐ Find the nth term from pattern sequences ➢ Mathswatch 103 (GCSE) ☐ Generate terms of a quadratic sequence when given the nth term ➢ Mathswatch 102 (GCSE) ☐ Find the nth term of a quadratic sequence ➢ Mathswatch 213 (GCSE)	**Geometric Sequence**	A sequence where each term is multiplied by a common number, known as the ratio, to get to the next term.
	Quadratic sequence	A sequence where the nth term has a largest power of n^2. The second difference will always be a constant value in a quadratic sequence.

Using position to term rules

These are often described using the nth term rule. This is just a rule with a letter n in it. The n is replaced by the position of the number in the sequence.

e.g. The nth term rule of a sequence is $3n + 4$. What are the first 4 numbers in the sequence?

For the first term, $n = 1$ as it is position 1 in the sequence. For the second term $n = 2$, the third term $n = 3$ and the 4^{th} term $n = 4$.

Remember 3n means n x 3, so if n = 1 that is 3 x 1

$n = 1$ $\quad 3 \times 1 + 4 = 7$

$n = 2$ $\quad 3 \times 2 + 4 = 10$

$n = 3$ $\quad 3 \times 3 + 4 = 13$

$n = 4$ $\quad 3 \times 4 + 4 = 16$

The first 4 terms are 7, 10, 13 and 16.

If we wanted the 100^{th} term we would use $n = 100$ and so on for any other position in the sequence.

Finding if a number is in a sequence

e.g. is 311 a term in the sequence $4n + 5$

To decide with questions like this, first set it up as an equation and then solve. If n is an integer at the end it is in the sequence and that is its position:

$$4n + 5 = 311 \quad (-5)$$
$$4n = 306 \quad (\div 4)$$
$$n = \frac{306}{4} = 76.5$$

No, 311 is not in the sequence as it is between the 76^{th} and 77^{th} term.

Finding position to term rules (nth term)

e.g. Find the nth term rule of the sequence 5, 8, 11, 14.....

+3 +3 +3

5, 8, 11, 14 ....

The sequence goes up by 3 each time so must be related to the 3 times table. The nth term of the 3x table is 3n.

Sequence $\quad$ 5, 8, 11, 14

+2

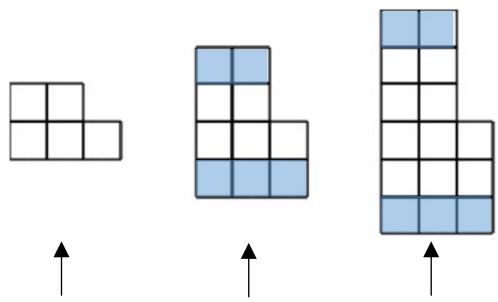
3x table $\quad$ 3, 6, 9, 12

To go from the 3 times table to the sequence we always add 2. So the nth term is $3n + 2$

Pattern Sequences

Often patterns of shapes can be simplified to a number sequence.

e.g.



n = 1 $\quad$ n = 2 $\quad$ n = 3

Each extra term adds 2 squares to the top and 3 squares to the bottom. In total it goes up by 5 squares each time.

The sequence in this case is the number of squares in each shape so the sequence is 5, 10, 15 ......

The nth term of this sequence would be $5n$.

Fibonacci sequence

The classic Fibonacci sequence starts 0, 1, 1, 2, 3, 5, 8, 13, 21 ...

After the first 2 terms, the next one is the sum of the 2 previous terms. So the next term would be 13 + 21 = 34.

Geometric sequences (or geometric progressions)

General facts:

To get from one term to the next you multiply by a constant value known as the ratio.

- If the ratio is larger than 1 the sequence increases.
- If the ratio is between 1 and 0 the sequence decreases.
- If the ratio is negative the sequence oscillates between positive and negative values.

e.g. First term is 5, ratio is 2

5, 10, 20, 40, 80, 160, ...

First term is 8, ratio is 0.5

8, 4, 2, 1, 0.5, 0.25, 0.125, ...

First term is 5, ratio is -3

5, -15, 45, -135, 405, ...

- To calculate r when given the sequence, divide any term in the sequence by the term before it.

e.g. The 5th term of a geometric sequence is 2500 and the fourth term is 500. What is the first term?

$$\_\_\_,\ \_\_\_,\ \_\_\_,\ 500 \xrightarrow{\times r} 2500, \ldots$$

2500 ÷ 500 = 5 so the ratio (r) = 5

500 ÷ 5 = 100 which is the third term

100 ÷ 5 = 20 which is the second term

20 ÷ 5 = 4 which is the first term

Nth term:

The general nth term of a geometric sequence is ar^{n-1} where a is the first term of the sequence and r is the ratio of the sequence.

e.g. For the sequence 4, 12, 36, 108, ...

a = 4 (as the first term is 4)   r = 3 (12 ÷ 4 = 3, 36 ÷ 12 = 3 and 108 ÷ 36 = 3)

So the nth term is $4 \times 3^{n-1}$

Checking this works for n = 2:

$4 \times 3^{2-1} = 4 \times 3^1 = 4 \times 3 = 12$ which matches our 2nd term

Quadratic Sequences

The simplest quadratic sequence is the list of square numbers and has the nth term of n^2

1, 4, 9, 16, 25, ...

The sequence of triangular numbers is also a quadratic sequence and has the nth term of $\frac{1}{2}n(n+1)$ or $\frac{1}{2}n^2 + \frac{1}{2}n$

1, 3, 6, 10, 15, 21, ...

These sequences have a different amount between each term but the difference between these, known as the second difference, is constant.

e.g. for the triangular numbers

Sequence	1		3		6		10		15		21
1st difference		+2		+3		+4		+5		+6	
2nd difference			+1		+1		+1		+1		

The second difference is always double the amount of n^2 in the nth term i.e. if you need to find the nth term you start by halving the second difference and using that as the coefficient of n^2.

Nth term of a quadratic sequence

- Find the coefficient of n^2
- Multiply the value of n^2 for each term by this coefficient and subtract from the original sequence
- Find the nth term of the remaining linear sequence.

e.g. Find the nth term of the sequence 5, 7, 11, 17, 25, ...

Not a linear sequence as the 1st difference is not constant.

Not a geometric sequence as there is not a constant ratio ($7 \div 5 = 1.4$ but $11 \div 7 = 1.57...$)

Sequence	5		7		11		17		25
1st difference		+2		+4		+6		+8	
2nd difference			+2		+2		+2		

As the second difference is 2, half of this gives us one lot of n^2

n	1	2	3	4	5
Sequence	5	7	11	17	25
n^2	1	4	9	16	25
Sequence minus n^2	4	3	2	1	0

The nth term of 4, 3, 2, 1, 0, ... is: $-n + 5$

This part is a linear sequence with a constant difference of -1 so we use a normal method for finding the nth term: see Position to Term Rules (nth term)

Therefore the nth term of the quadratic sequence is: $n^2 - n + 5$

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