

Y10 Maths Knowledge Organiser Higher Tier: Quadratics Equations

What must I be able to do?	Key vocabulary	
New content: <ul style="list-style-type: none"> □ Solve a quadratic equation by factorising <ul style="list-style-type: none"> ➤ Mathswatch 157 (GCSE) □ Solve a quadratic equation by using the quadratic formula <ul style="list-style-type: none"> ➤ Mathswatch 191 (GCSE) □ Solve a quadratic equation by completing the square <ul style="list-style-type: none"> ➤ Mathswatch 209a and 209b (GCSE) □ Identify the significant points of a quadratic function <ul style="list-style-type: none"> ➤ Mathswatch 209c (GCSE) □ Solve a pair of simultaneous equations where one is non linear using an algebraic method <ul style="list-style-type: none"> ➤ Mathswatch 211 (GCSE) □ Solve quadratic inequalities <ul style="list-style-type: none"> ➤ Mathswatch 212 (GCSE) 	Root	The values of x in a quadratic equation which give a value of $y = 0$. On a graph, this is where it <u>crosses the x-axis</u> .
	Turning point	On a quadratic graph, the turning point is the <u>maximum or minimum</u> point on the curve.
	Discriminant	The part of the formula <u>under the square root</u> ($b^2 - 4ac$). It determines how many solutions a quadratic equation will have.

Solving by factorising

Step 1: Rearrange the equation so that one side is equal to 0

Step 2: Factorise the equation

Step 3: Solve each factor equal to 0.

e.g. Solve $x^2 - 6x + 10 = 2$

$$x^2 - 6x + 8 = 0$$

$$(x - 4)(x - 2) = 0$$

Either $x - 4 = 0$ or $x - 2 = 0$

$$x = 4 \quad \text{and} \quad x = 2$$

e.g. Solve $2x^2 - 5x - 3 = 0$

$$(2x + 1)(x - 3) = 0$$

Either $2x + 1 = 0$ or $x - 3 = 0$

$$2x = -1$$

$$x = -\frac{1}{2} \quad \text{and} \quad x = 3$$

The quadratic formula

For a general quadratic equation written $ax^2 + bx + c = 0$

$$\text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

e.g. Solve $4x^2 - 8x - 7 = 0$

$$a = 4 \quad b = -8 \quad c = -7$$

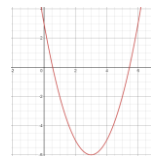
$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \times 4 \times (-7)}}{2 \times 4}$$

$$x = 2.66 \quad \text{and} \quad x = -0.66 \quad (2.d.p.)$$

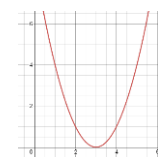
Be careful putting negatives into your calculator. Brackets around the negative number will help.

The $b^2 - 4ac$ is known as the **discriminant**.

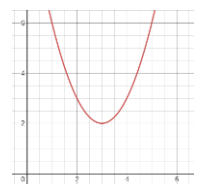
If $b^2 - 4ac > 0$ then there are 2 unique roots. The graph crosses the x-axis in 2 places.



If $b^2 - 4ac = 0$ then there is a repeated root. The graph touches the x-axis in one spot.



If $b^2 - 4ac < 0$ there are no roots. The graph does not touch the x-axis.



Completing the square

Writing a quadratic equation in the form $(x + p)^2 + r = 0$ is known as completing the square.

e.g. Solve $x^2 + 6x - 8 = 0$

$(x + 3)^2 - 9 - 8 = 0$

(Half the b value, so $6 \div 2 = 3$)

(subtract this value squared as $(x + 3)^2$ multiplied out is $x^2 + 6x + 9$, not $x^2 + 6x$)

$$(x + 3)^2 - 17 = 0 \quad (\text{this is the completed the square form})$$

$$(x + 3)^2 = 17$$

$$x + 3 = \pm\sqrt{17}$$

$$x = -3 \pm \sqrt{17}$$

When written in the form $(x + p)^2 + r = 0$, you can determine key features of the graph.

The equation of the **line of symmetry** of the curve is $x = -p$

The co-ordinate of the **turning point** of the curve (minimum/maximum point) is $(-p, r)$

Solving quadratic inequalities

Solving a quadratic inequality is very similar to solving a quadratic equation.

Step 1: Solve the equation to find the critical values.

Step 2: Sketch the curve

Step 3: Write down the appropriate inequality/inequalities

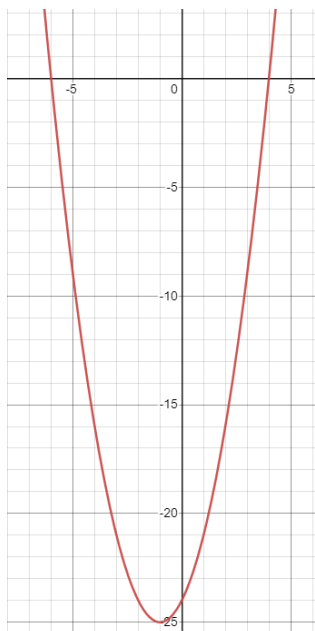
e.g. Solve $x^2 + 10x - 24 < 0$

Start by solving:

$$x^2 + 10x - 24 = 0$$

$$(x + 6)(x - 4) = 0$$

$$x = -6 \text{ and } x = 4 \text{ (these are the critical values)}$$



The curve is a positive quadratic so is a 'u' shaped parabola.

The roots of the equation are at $x = -6$ and $x = 4$, so this is where it crosses the x axis.

The curve is < 0 (below the x-axis) when it is between $x = -6$ and $x = 4$.

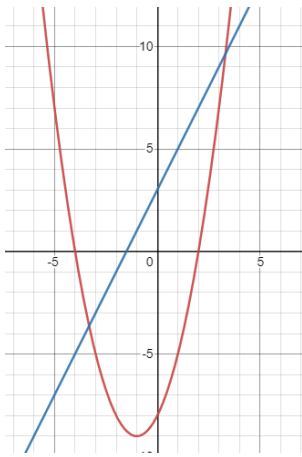
Therefore the solution to $x^2 + 10x - 24 < 0$ is:

$$-6 < x < 4$$

Note: if the question instead was solve $x^2 + 10x - 24 > 0$ we now need the sections above the x-axis which are not connected and so the solution would have been

$$x < -6 \text{ and } x > 4$$

Simultaneous equations where one is non-linear



As a non-linear graph will curve, the solution to simultaneous equations with a non-linear equation can have more than 1 answer.

If we are solving a quadratic and a linear graph there are either:

0 solutions – the graphs do not intersect

1 solution – the linear graph is a tangent to the curve and touches only once

2 solutions – the graph crosses twice (as shown on the left)

Solving the equations algebraically allows us to find the exact values of these intersections.

e.g. Solve $y = x^2 + 3x - 8$
 $y = 2x + 3$

As both equations are $y =$, we can equate them

$$x^2 + 3x - 8 = 2x + 3$$

Rearrange so that one side = 0

$$x^2 + x - 11 = 0$$

This does not factorise so using the formula

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-11)}}{2 \times 1}$$

$$x = 2.8541... \text{ and } x = -3.8541...$$

Substitute these back into $y = 2x + 3$

$$y = 8.7082... \text{ and } y = -4.7082...$$

So the solutions are:

$$x = 2.86 \text{ and } y = 8.71$$

$$x = -3.85 \text{ and } y = -4.71$$

e.g. Solve $x^2 + y^2 = 10$ ↖ This is the equation of a circle
 $y = 2x - 5$

This time we need to substitute $y = 2x - 5$ into the top equation.

$$x^2 + (2x - 5)^2 = 10$$

Multiply out the bracket

$$x^2 + 4x^2 - 20x + 25 = 10$$

Simplify and set one side = 0

$$5x^2 - 20x + 15 = 0$$

Factorise and solve

$$5(x^2 - 4x + 3) = 0$$

$$5(x - 3)(x - 1) = 0$$

$$x = 3 \text{ and } x = 1$$

Substitute back into $y = 2x - 5$

$$\text{When } x = 3, y = 1$$

$$\text{When } x = 1, y = -3$$

Solutions need to be given in pairs with the correct x and y values matched up.

GLUE

HERE