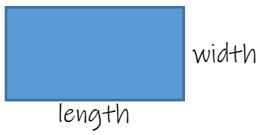
Y10 Maths Knowledge Organiser Higher Tier: Length, Area and Volume

What must I be able to do?	Key vocabulary	
New content: ☐ Calculate the length of an arc ➢ Mathswatch 167 (GCSE) ☐ Calculate the area and angle of a sector ➢ Mathswatch 167 (GCSE) ☐ Calculate the volume of a pyramid ➢ Mathswatch 170 (GCSE) ☐ Calculate the volume and surface area of a cone ➢ Mathswatch 171 (GCSE) ☐ Calculate the volume and surface area of a sphere ➢ Mathswatch 169 (GCSE)	**Sector**	A fraction of a circle, cut from the centre like a slice of pizza. The two straight sides will be the radius of the circle.
	Arc	A section of the circumference of a circle.
	Frustum	The remaining shape when the top of a cone or pyramid is cut off at an angle parallel to it's base.

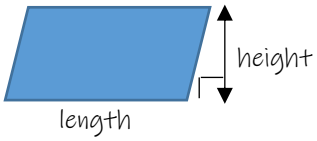
Area formulae

Rectangle/Square



Area = Length x width

Parallelogram



Area = length x perpendicular height

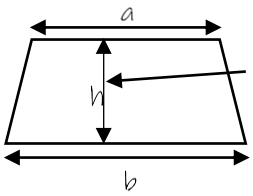
Triangles

Area = Base x perpendicular height ÷ 2

Perpendciular means at right angles to the base (not the sloping side!)

A triangle is half the area of a rectangle

Trapezium

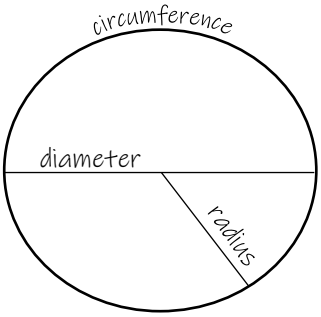


Height must be perpendicular to the parallel sides.

$$\text{Area} = \frac{1}{2}(a + b)h$$

$\frac{1}{2}(a + b)$ finds the average length of the parallel sides. This essentially turns the formula into the same as for the area of a parallelogram!

Circles



The **area** of a circle is equal to π multiplied by the radius squared :

$$A = \pi r^2$$

Note that just the r is squared, not π

Rearranging this gives us:

$$r = \sqrt{\frac{A}{\pi}}$$

The **circumference** of a circle is equal to π multiplied by the diamater:

$$C = \pi d$$

Converting units of area

When converting units of area, you need to do the standard length conversion rule **twice**, once for each dimension.

$1m^2 = 1m \times 1m = 100cm \times 100cm = 10{,}000cm^2$

$1cm^2 = 1cm \times 1cm = 10mm \times 10mm = 100mm^2$

Therefore $1m^2 = 1{,}000{,}000mm^2$

Converting units of volume

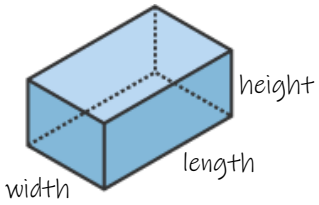
Do the length conversion **three** times, once for each dimension.

$1\ m^3 = 1m \times 1m \times 1m = 100cm \times 100cm \times 100cm = 1{,}000{,}000\ cm^3$

$1\ cm^3 = 1cm \times 1cm \times 1cm = 10mm \times 10mm \times 10mm = 1{,}000\ mm^3$

$1000\ cm^3 = 1$ litre so $1\ m^3 = 1000$ litres

Cubes/cuboids



Volume = length x width x height

Surface area:

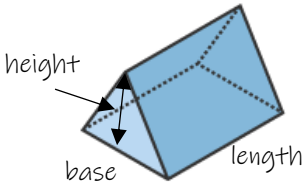
Front + back: length x height x 2 (rectangles)

Side + side = width x height x 2 (rectangles)

Top + bottom = length x width x 2 (rectangles)

Total surface area is these 3 added together.

Triangular prisms



$$\text{Volume} = \frac{\text{base x perpendicular height}}{2} \text{ x length}$$

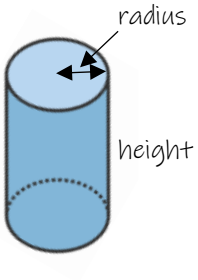
Surface area:

Area of the 2 triangles ($\frac{b \times h}{2}$ for each one)

Area of the three rectangles (note that they may all be different!)

Total surface area is all 5 faces added together.

Cylinders



Volume = π x radius squared x height

$= \pi r^2 h$

Surface area:

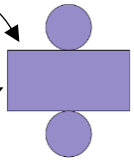
Top + bottom: Area of circle x 2

Curved surface area = area of rectangle

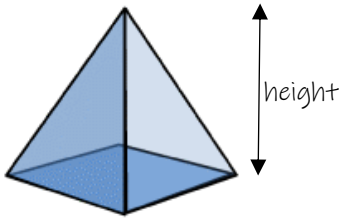
Total surface area is both added together.

$S.A = 2\pi r^2 + 2\pi rh$

The curved surface area is the rectangular part of the net of a cylinder. It has a length equal to the circumference of the circle at the top of the cylinder and a height equal to that of the cylinder.



Pyramids

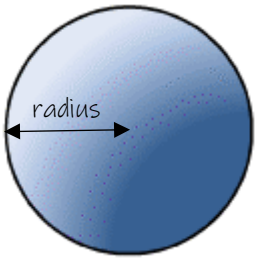


Volume $= \frac{1}{3} \times$ area of base $\times$ perpendicular height

Surface area = area of base + area of all the triangles

Given to you in an exam!

Spheres



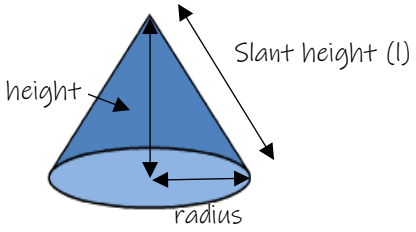
Volume $= \frac{4}{3} \times \pi \times$ radius cubed

$= \frac{4}{3}\pi r^3$

Surface area = 4 x π x radius squared

$= 4\pi r^2$

Cones



Volume = $\frac{1}{3}$ x pi x radius squared x height

$= \frac{1}{3}\pi r^2 h$

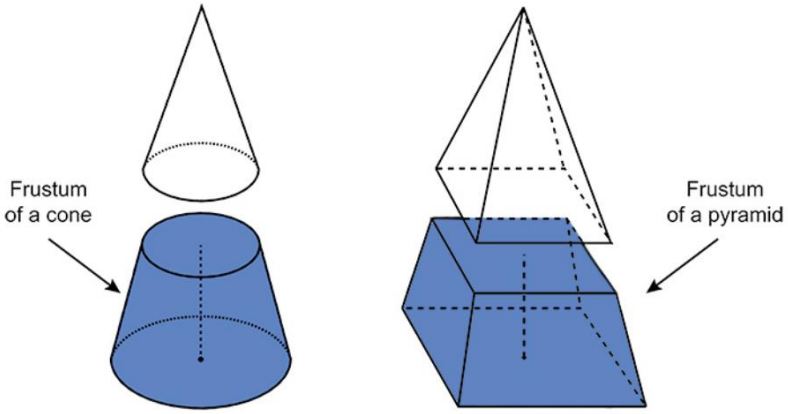
Given to you in an exam!

Curved surface area = pi x radius x slant height

$= \pi rl$

Total surface area = $\pi r^2 + \pi rl$

Frustums



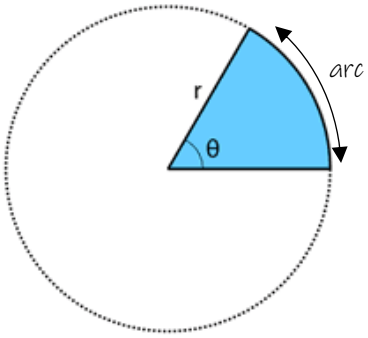
When removing the top section of a cone or pyramid the remaining shape is known as a frustum.

Key fact: the shape which is removed is a **similar** shape to the original one, i.e. it is a **scale factor enlargement of the original shape** (usually a fraction e.g. ½)

This means that the radius/side lengths of the original shape and the section removed will have the same ratio of lengths as the heights do

Volume of a frustum = volumeof original shape – volume of shape removed

Arcs and sectors



Area of a sector = fraction of a full circle x area of a circle

$= \frac{\theta}{360}\pi r^2$

Arc length = fraction of a full circle x circumference

$= \frac{\theta}{360}\pi d$

Perimeter of a sector = arc length + radius + radius

$= \frac{\theta}{360}\pi d + 2r$

GLUE

HERE