Y10 Maths Knowledge Organiser Higher Tier: Right Angled Triangles

What must I be able to do?

New content:

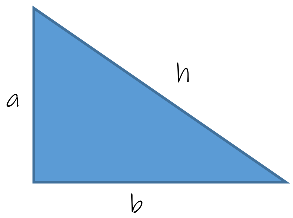
- ☐ Use Pythagoras' theorem to find a missing side in a right angled triangle
  - ➢ Mathswatch 150a and 150b (GCSE)
- ☐ Use Pythagoras' theorem to solve problems, including 3D
  - ➢ Mathswatch 150c and 217 (GCSE)
- ☐ Use the three trigonometric ratios to find a missing side
  - ➢ Mathswatch 168 (GCSE)
- ☐ Use the trig ratios to calculate an angle
  - ➢ Mathswatch 168 (GCSE)
- ☐ Solve practical problems using trigonometry, including bearings and angles of elevation and depression
- ☐ Know certain values for exact trig functions
  - ➢ Mathswatch 173 (GCSE)
- ☐ Use trigonometry in 3D
  - ➢ Mathswatch 218 (GCSE)

Key vocabulary

Hypotenuse	The longest side of a right angled triangle. It is the side opposite the right angle.
Angle of elevation	The angle made with the ground by looking up at something.
Angle of depression	The angle made with the ground by looking down at something e.g. from the top of a cliff or tower.
Plane	Flat, 2 dimensional surface

Pythagoras' Theorem

Pythagoras' theorem states that in a right angled triangle, **the square of the hypotenuse is equal to the sum of the squares of the other two sides.**



$h^2 = a^2 + b^2$

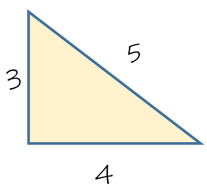
so therefore by rearranging we also get:

$a^2 = h^2 - b^2$ and

$b^2 = h^2 - a^2$

Pythagorean Triples

These are sets of 3 integer values which form a right angled triangle



The most common Pythagorean triple is the **3, 4, 5** triangle

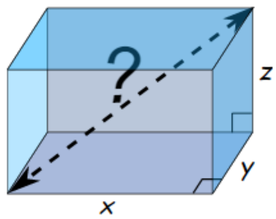
Any integer scale factor enlargement of a Pythagorean triple also gives another triple

e.g. 3, 4, 5 can become 6, 8, 10 (s.f. 2) or 9, 12, 15 (s.f. 3)

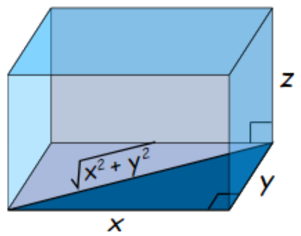
The next 6 primitive (non enlarged) Pythagorean triples are:

5, 12, 13	9, 40, 41
7, 24, 25	11, 60, 61
8, 15, 17	12, 35, 37

Pythagoras in 3D



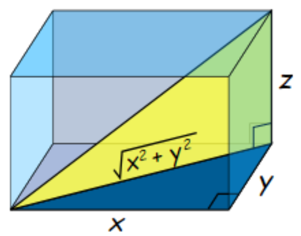
In order to find the diagonal distance across the cuboid, we would need to use Pythagoras' theorem twice



Working on the base of the cuboid:

$h^2 = x^2 + y^2$

$h = \sqrt{x^2 + y^2}$



Now with the diagonal across the cuboid

$h^2 = \sqrt{x^2 + y^2}^2 + z^2$

$h^2 = \sqrt{x^2 + y^2 + z^2}$

So to find the length of a diagonal in 3 dimensions we can extend Pythagoras' Theorem to include all 3 side lengths

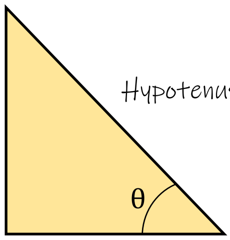
$$h^2 = \sqrt{x^2 + y^2 + z^2}$$

Trigonometric Ratios

For any right angled triangle, if we identify one angle we can label the 3 sides as shown

Opposite (O)

Opposite the marked angle



Hypotenuse (H)

Adjacent (A)

Next to the marked angle

The ratio of each pair of the 3 sides, is always the same answer for a given size of the angle θ, regardless of the actual lengths of the sides.

This leads to the following definitions:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

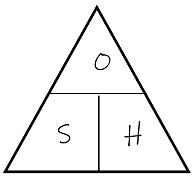
Sin is short for sine, cos for cosine and tan for tangent.

One way to remember these is the mnemonic **SOHCAHTOA** which gives each of the 3 ratios by their first letter.

We can also represent these ratios using formula triangles. In each case the letter in the middle goes at the top of the triangle

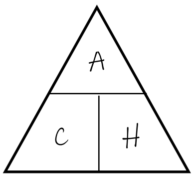
SOH

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$



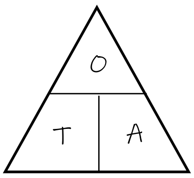
CAH

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

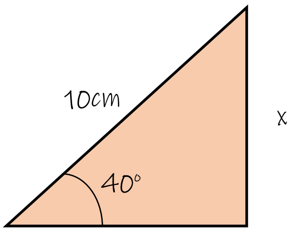


TOA

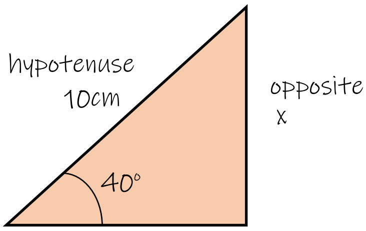
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$



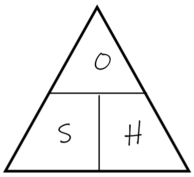
Using trigonometry to find a missing side



Start by labelling the two sides in the question



The ratio with O and H in is sine

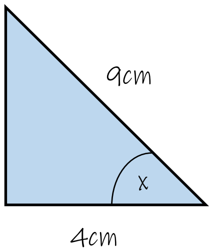


$$\sin 40 = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{x}{10}$$

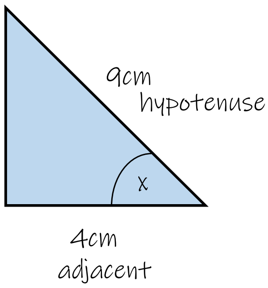
$$10 \times \sin 40 = x$$

$$x = 6.43 \text{ (2.d.p)}$$

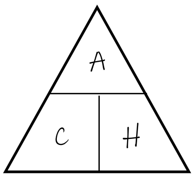
Using trigonometry to find a missing angle



Start by labelling the two sides in the question



The ratio with A and H in is cosine



$$\cos x = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{9}$$

$$x = \cos^{-1}\left(\frac{4}{9}\right)$$

$$x = 63.6^\circ$$

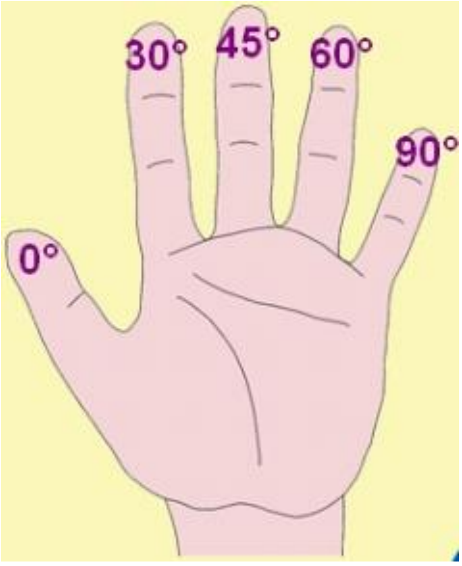
The **inverse** of each trig function is written as

$\sin^{-1} x$ $\cos^{-1} x$ and $\tan^{-1} x$

Use these when finding an angle

Exact trig values for 0, 30, 45, 60 and 90°

On a **non-calculator** paper you can be asked to complete a trigonometry question if the angle is 0, 30, 45, 60 or 90°. Therefore you need to learn the following standard values for these angles.



To find **sine** of one of these 5 angles, identify the correct finger on your left hand. Square root the number of fingers held up to the **left** of that finger and then divide by 2 to get an exact value for the sine of that angle.

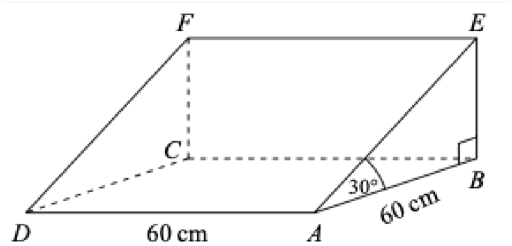
The **cosine** is the square root of the number of fingers to the **right** of that finger and then divide by 2.

The **tangent** is the square root of the fraction of the number of fingers to the left (sine), divided by the number of fingers to the right (cosine).

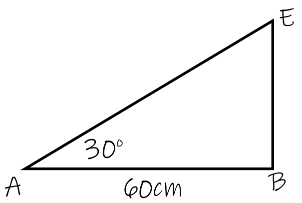
	0	30	45	60	90
sin	$\frac{\sqrt{0}}{2}=0$	$\frac{\sqrt{1}}{2}=\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}=\frac{2}{2}=1$
cos	$\frac{\sqrt{4}}{2}=\frac{2}{2}=1$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}=\frac{1}{2}$	$\frac{\sqrt{0}}{2}=0$
tan	$\sqrt{\frac{0}{4}}=0$	$\sqrt{\frac{1}{3}}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$	$\sqrt{\frac{2}{2}}=\frac{\sqrt{2}}{\sqrt{2}}=1$	$\sqrt{\frac{3}{1}}=\frac{\sqrt{3}}{\sqrt{1}}=\sqrt{3}$	Does not exist

Trigonometry in 3D

Example

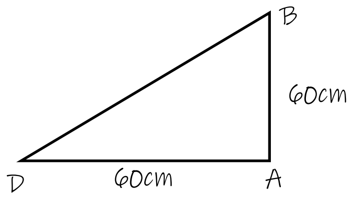


$AEFD$ is a rectangle. $ABCD$ is a square.
EB and FC are perpendicular to plane $ABCD$. $AB = 60$ cm. $AD = 60$ cm. Angle $ABE = 90°$. Angle $BAE = 30°$. Calculate the size of the angle that the line DE makes with the plane $ABCD$.
Give your answer correct to 1 decimal place.



$$\tan 30 = \frac{BE}{60}$$

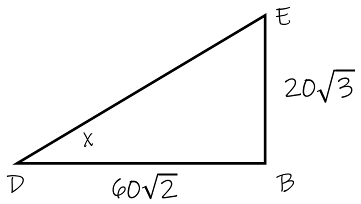
$$BE = 60 \times \tan 30 = 20\sqrt{3}$$



$$BD^2 = 60^2 + 60^2$$

$$BD^2 = 7200$$

$$BD = \sqrt{7200} = 60\sqrt{2}$$



$$\tan x = \frac{20\sqrt{3}}{60\sqrt{2}}$$

$$x = \tan^{-1}\frac{20\sqrt{3}}{60\sqrt{2}} = 22.20765 = 22.2^0$$

To solve problems like this, split the question into 2D right angled triangles and apply Pythagoras' theorem and/or trigonometry.

There is more than one approach to solving this question and we could have found side AE, then side DE instead of BE and BD.

GLUE

HERE