Y10 Maths Knowledge Organiser Higher Tier: Rational and Irrational Numbers

What must I be able to do?

New content:

- ☐ Convert recurring decimals to fractions
  - ➢ Sparx U689, U550
- ☐ Use rules of negative and fractional powers
  - ➢ Sparx U985, U772
- ☐ Simplify surds
  - ➢ Sparx U338, U633, U872, U499
- ☐ Manipulate surds including rationalising a denominator
  - ➢ Sparx U707, U281
- ☐ Find the limits of accuracy of rounded numbers
  - ➢ Sparx U587, U657
- ☐ Combine the bounds of two or more variables to solve problems
  - ➢ Sparx U375
- ☐ Calculate permutations and combinations

Key vocabulary

Recurring decimal	A decimal in which a figure or group of figures is repeated indefinitely e.g. 0.6666666....
Surd	A number inside a square root (or cube root etc) which cannot be further simplified.
Lower bound	The lower bound is the smallest value that would round up to the estimated value.
Upper bound	The upper bound is the smallest value that would round up to the next estimated value.
Combinations	The number of ways we can choose something when the order does not matter.
Permuations	The number of ways we can choose something where the order does matter.

Recurring decimals to fractions

Manipulate the decimal so that the recurring part is the only bit after the decimal point in 2 different expressions and then subtract them from each other.

e.g. Convert $0.6\dot{3}$ into a fraction in its simplest form.

$0.6\dot{3}$ is 0.6333333.....

Let $x = 0.6\dot{3}$

$10x = 6.\dot{3}$ (Equation 1)

$100x = 63.\dot{3}$ (Equation 2)

Equation 2 – equation 1

$90x = 57$

$$x = \frac{57}{90} = \frac{19}{30}$$

e.g. Convert $0.\dot{7}\dot{2}$ into a fraction in its simplest form.

$0.\dot{7}\dot{2}$ is 0.72727272....

Let $x = 0.\dot{7}\dot{2}$ (Equation 1)

$100x = 72.\dot{7}\dot{2}$ (Equation 2)

Equation 2 – Equation 1

$99x = 72$

$$x = \frac{72}{99} = \frac{8}{11}$$

Basic power rules (recap from Y9 topic 2)

$a^0 = 1$

$a^1 = a$

$a^b \times a^c = a^{b+c}$

$a^b \div a^c = a^{b \div c}$

$(a^b)^c = a^{b \times c}$

Negative Powers

A number raised to a negative power is the **reciprocal** of that number raised to the positive power.

$$x^{-4} = \frac{1}{x^4}$$

e.g. $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

e.g. $\left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^4 = \frac{3^4}{2^4} = \frac{81}{16}$

Fractional Powers

The numerator of a fractional power is an ordinary power.

The denominator of a fractional power represents a root.

$$a^{\frac{x}{y}} = \sqrt[y]{a^x} = (\sqrt[y]{a})^x$$

e.g. $16^{\frac{3}{4}} = \sqrt[4]{16}^3 = 2^3 = 8$

Simplifying Surds

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$$

Any coefficients infront of the surds are dealt with separately.

e.g. $2\sqrt{5} \times 3\sqrt{4} = 2 \times 3 \times \sqrt{5 \times 4} = 6\sqrt{20}$

We can use this in reverse to simplify the value inside the surd. Take out a factor of the surd which is a **square number** as this can be square rooted.

e.g. $\sqrt{48} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$

Check your final answer to see if it can be further simplified. This will happen if you do not spot the largest factor when simplifying (similar to fractions).

e.g.

$$\begin{aligned}\sqrt{72} &= \sqrt{9} \times \sqrt{8} \\ &= 3\sqrt{8} \\ &= 3\sqrt{4} \times \sqrt{2} \\ &= 3 \times 2 \times \sqrt{2} \\ &= 6\sqrt{2}\end{aligned}$$

8 has a factor of 4 which is square

Surds of the **same value** can be **added and subtracted**

e.g.

$$5\sqrt{3} + 4\sqrt{3} = 9\sqrt{3}$$

Brackets and Surds

You can expand brackets with surds in them using normal methods

e.g. $\sqrt{5}(3+\sqrt{3}) = 3 \times \sqrt{5} + \sqrt{3} \times \sqrt{5}$

$= 3\sqrt{5} + \sqrt{15}$

For a double bracket, use whichever method you would normally use e.g. partitioning, multiplication grid, smiley face, etc.

e.g. $(3+\sqrt{5})(4-\sqrt{5}) =$

$3 \times 4 = 12$

$3 \times -\sqrt{5} = -3\sqrt{5}$

$\sqrt{5} \times 4 = 4\sqrt{5}$

$4\sqrt{5} - 3\sqrt{5} = \sqrt{5}$

$\sqrt{5} \times \sqrt{5} = \sqrt{25} = 5$

$= 17 + \sqrt{5}$

Rationalising Surds

Rationalising a surd means to not have an irrational number on the denominator of the fraction. As most surds are irrational numbers we need to remove any surds from the denominator.

In order to achieve these we need to choose an appropriate value to multiply both the numerator and denominator by which will cancel out the surd.

If there is only a single term on the denominator, multiply by the surd which is on the denominator.

e.g. Rationalise $\frac{4}{3\sqrt{5}}$

$$\begin{aligned}\frac{4}{3\sqrt{5}} &= \frac{4}{3\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{4 \times \sqrt{5}}{3\sqrt{5} \times \sqrt{5}} \\ &= \frac{4\sqrt{5}}{3 \times 5} \\ &= \frac{4\sqrt{5}}{15}\end{aligned}$$

If there are 2 terms on the denominator, multiply by the difference of two squares and simplify.

e.g. Rationalise $\frac{3+\sqrt{2}}{2-\sqrt{5}}$

$$\begin{aligned}\frac{3+\sqrt{2}}{2-\sqrt{5}} &= \frac{3+\sqrt{2}}{2-\sqrt{5}} \times \frac{2+\sqrt{5}}{2+\sqrt{5}} \\ &= \frac{(3+\sqrt{2})(2+\sqrt{5})}{(2-\sqrt{5})(2+\sqrt{5})} \\ &= \frac{6+2\sqrt{2}+3\sqrt{5}+\sqrt{10}}{4-2\sqrt{5}+2\sqrt{5}-5} \\ &= \frac{6+2\sqrt{2}+3\sqrt{5}+\sqrt{10}}{-1} \\ &= -6-2\sqrt{2}-3\sqrt{5}-\sqrt{10}\end{aligned}$$

Tip:

The need to rationalise a surd is usually a non-calculator question as most modern scientific calculators will rationalise a surd for you just by inputting it and pressing '='. Should you need to do it on a calculator paper you can normally just use your calculator, however, a complicated one such as the second example here may not display properly and instead be given as a decimal!

Error intervals

In the real world, many numbers, especially measurements, have been rounded to a specific degree of accuracy. Common examples would be to the nearest whole number, a number of decimal places or a number of significant figures. An error interval displays all the possible values a number could be between, which would round to the specificed value.

e.g. State the error interval if the length of a pencil is 12cm rounded to the nearest cm.

The smallest value which would round to 12cm to the nearest whole cm, is half a cm smaller than 12 which is 11.5cm.

The smallest value which would round to 13cm to the nearest whole cm, is half a cm larger than 12 which is 12.5cm.

Therefore the error interval is $11.5 \leq 12 < 12.5$ Note that this **includes 11.5** but **not include 12.5.**

e.g. State the error interval if the height of a man is 1.8m to 1.d.p.

The smallest value which rounds to 1.8m to 1.d.p. is half a tenth smaller than 1.8m and is 1.75m.

The smallest value which rounds to 1.9m to 1.d.p. is half a tenth larger than 1.8m and is 1.85m.

Therefore the error interval is $1.75 \leq 1.8\text{m} < 1.85\text{m}$.

Truncation

Sometimes numbers are not rounded but are instead truncated (cut off).

e.g. 2.345678 could be truncated to 2, or 2.3, or 2.34, or 2.345 etc. No values are rounded up.

e.g. A number has been truncated to 4.8. State the error interval.

The smallest number this could have been is 4.80000.... = 4.8.

The largest number this could have been is 4.899999.... ≈ 4.9

Therefore the error interval is $4.8 \leq 4.8 < 4.9$

Working with bounds/limits

Problem solving questions involving limits of accuracy generally require calculations to find a minimum or maximum possible value.

It depends which function you need to calculate as to whether you need to use the minimum or maximum limits in your working.

e.g. For 2 values X and Y which have been rounded to a specific accuracy, they can each be written as an error interval of

$$A \leq X < B \quad \text{and} \quad C \leq Y < D$$

Calculation	Minimum answer	Maximum answer
$X + Y$	$A + C$ (smallest + smallest)	$B + D$ (largest + largest)
$X - Y$	$A - D$ (smallest – largest)	$B - C$ (largest – smallest)
$X \times Y$	$A \times C$ (smallest × smallest)	$B \times D$ (largest × largest)
$X \div Y$	$A \div D$ (smallest ÷ largest)	$B \div C$ (largest ÷ smallest)

GLUE

HERE