

Y10 Maths Knowledge Organiser Higher Tier: Sequences

What must I be able to do?	Key vocabulary				
<p>New content:</p> <ul style="list-style-type: none"> □ Know and use geometric sequences <ul style="list-style-type: none"> ➤ Sparx U958 □ Find the nth term from pattern sequences <ul style="list-style-type: none"> ➤ Sparx M866 □ Generate terms of a quadratic sequence when given the nth term <ul style="list-style-type: none"> ➤ Sparx U530 □ Find the nth term of a quadratic sequence <ul style="list-style-type: none"> ➤ Sparx U206 	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 30%;">Geometric Sequence</td> <td>A sequence where each term is multiplied by a common number, known as the ratio, to get to the next term.</td> </tr> <tr> <td>Quadratic sequence</td> <td>A sequence where the nth term has a largest power of n^2. The second difference will always be a constant value in a quadratic sequence.</td> </tr> </table>	Geometric Sequence	A sequence where each term is multiplied by a common number, known as the ratio, to get to the next term.	Quadratic sequence	A sequence where the nth term has a largest power of n^2 . The second difference will always be a constant value in a quadratic sequence.
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Using position to term rules

These are often described using the nth term rule. This is just a rule with a letter n in it. The n is replaced by the position of the number in the sequence.

e.g. The nth term rule of a sequence is $3n + 4$. What are the first 4 numbers in the sequence?

For the first term, $n = 1$ as it is position 1 in the sequence. For the second term $n = 2$, the third term $n = 3$ and the 4th term $n = 4$.

$$n = 1 \quad 3 \times 1 + 4 = 7$$

$$n = 2 \quad 3 \times 2 + 4 = 10$$

$$n = 3 \quad 3 \times 3 + 4 = 13$$

$$n = 4 \quad 3 \times 4 + 4 = 16$$

Remember $3n$ means $n \times 3$, so if $n = 1$ that is 3×1

The first 4 terms are 7, 10, 13 and 16.

If we wanted the 100th term we would use $n = 100$ and so on for any other position in the sequence.

Finding if a number is in a sequence

e.g. is 311 a term in the sequence $4n + 5$

To decide with questions like this, first set it up as an equation and then solve. If n is an integer at the end it is in the sequence and that is its position:

$$\begin{array}{r}
 -5 \quad \left\{ \begin{array}{l} 4n + 5 = 311 \\ 4n = 306 \end{array} \right. \quad -5 \\
 \div 4 \quad \left\{ \begin{array}{l} n = \frac{306}{4} \\ n = 76.5 \end{array} \right. \quad \div 4
 \end{array}$$

No, 311 is not in the sequence as it is between the 76th and 77th term.

Finding position to term rules (nth term)

e.g. Find the nth term rule of the sequence 5, 8, 11, 14, ...

$$\begin{array}{c}
 +3 \quad +3 \quad +3 \\
 \curvearrowright \quad \curvearrowright \quad \curvearrowright \\
 5, 8, 11, 14, \dots
 \end{array}$$

The sequence goes up by 3 each time so must be related to the 3 times table. The nth term of the 3x table is $3n$.

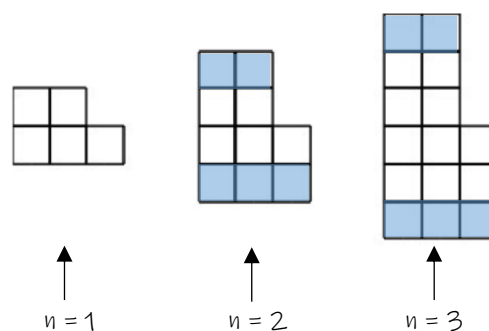
Sequence 5, 8, 11, 14
 3x table 3, 6, 9, 12

To go from the 3 times table to the sequence we always add 2. So the nth term is $3n + 2$

Pattern Sequences

Often patterns of shapes can be simplified to a number sequence.

e.g.



Each extra term adds 2 squares to the top and 3 squares to the bottom. In total it goes up by 5 squares each time.

The sequence in this case is the number of squares in each shape so the sequence is 5, 10, 15, ...

The nth term of this sequence would be $5n$.

Quadratic Sequences

The simplest quadratic sequence is the list of square numbers and has the n th term of n^2

1, 4, 9, 16, 25, ...

The sequence of triangular numbers is also a quadratic sequence and has the n th term of $\frac{1}{2}n(n+1)$ or $\frac{1}{2}n^2 + \frac{1}{2}n$

1, 3, 6, 10, 15, 21, ...

These sequences have a different amount between each term but the difference between these, known as the second difference, is constant.

e.g. for the triangular numbers

Sequence	1	3	6	10	15	21
1 st difference		+2	+3	+4	+5	+6
2 nd difference			+1	+1	+1	

The second difference is always double the amount of n^2 in the n th term i.e. if you need to find the n th term you start by halving the second difference and using that as the coefficient of n^2 .

Nth term of a quadratic sequence

- Find the coefficient of n^2
- Multiply the value of n^2 for each term by this coefficient and subtract from the original sequence
- Find the n th term of the remaining linear sequence.

e.g. Find the n th term of the sequence 5, 7, 11, 17, 25, ...

Not a linear sequence as the 1st difference is not constant.

Not a geometric sequence as there is not a constant ratio ($7 \div 5 = 1.4$ but $11 \div 7 = 1.57...$)

Sequence	5	7	11	17	25
1 st difference		+2	+4	+6	+8
2 nd difference			+2	+2	

As the second difference is 2, half of this gives us one lot of n^2

n	1	2	3	4	5
Sequence	5	7	11	17	25
n^2	1	4	9	16	25
Sequence minus n^2	4	3	2	1	0

The n th term of 4, 3, 2, 1, 0, ... is: $-n + 5$

This part is a linear sequence with a constant difference of -1 so we use a normal method for finding the n th term: see Position to Term Rules (n th term)

Therefore the n th term of the quadratic sequence is: $n^2 - n + 5$

GLUE

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