Y10 Maths Knowledge Organiser Higher Tier: Quadratics Equations

What must I be able to do?	Key vocabulary	
New content: ☐ Solve a quadratic equation by factorising ➢ Sparx U228, U960 ☐ Solve a quadratic equation by using the quadratic formula ➢ Sparx U665 ☐ Solve a quadratic equation by completing the square ➢ Sparx U589 ☐ Identify the significant points of a quadratic function ➢ Sparx U769, U667 ☐ Solve a pair of simultaneous equations where one is non linear using an algebraic method ➢ Sparx U547 ☐ Solve quadratic inequalities ➢ Sparx U133	**Root**	The values of x in a quadratic equation which give a value of y = 0. On a graph, this is where it <u>crosses the x-axis</u>.
	Turning point	On a quadratic graph, the turning point is the <u>maximum or minimum</u> point on the curve.
	Discriminant	The part of the formula <u>under the square root</u> ($b^2 - 4ac$). It determines how many solutions a quadratic equation will have.

Solving by factorising

Step 1: Rearrange the equation so that one side is equal to 0

Step 2: Factorise the equation

Step 3: Solve each factor equal to 0.

e.g. Solve $x^2 - 6x + 10 = 2$

$x^2 - 6x + 8 = 0$

$(x - 4)(x - 2) = 0$

Either $x - 4 = 0$ or $x - 2 = 0$

$x = 4$ and $x = 2$

e.g. Solve $2x^2 - 5x - 3 = 0$

$(2x + 1)(x - 3) = 0$

Either $2x + 1 = 0$ or $x - 3 = 0$

$2x = -1$

$x = -\frac{1}{2}$ and $x = 3$

The quadratic formula

For a general quadratic equation written $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

e.g. Solve $4x^2 - 8x - 7 = 0$

$a = 4 \quad b = -8 \quad c = -7$

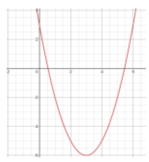
Be careful putting negatives into your calculator. Brackets around the negative number will help.

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \times 4 \times (-7)}}{2 \times 4}$$

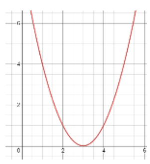
$x = 2.66$ and $x = -0.66$ (2.d.p.)

The $b^2 - 4ac$ is known as the **discriminant.**

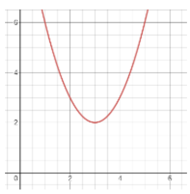
If $b^2 - 4ac > 0$ then there are 2 unique roots. The graph crosses the x-axis in 2 places.



If $b^2 - 4ac = 0$ then there is a repeated root. The graph touches the x-axis in one spot.



If $b^2 - 4ac < 0$ there are no roots. The graph does not touch the x-axis.



Completing the square

Writing a quadratic equation in the form $(x + p)^2 + r = 0$ is known as completing the square.

e.g. Solve $x^2 + 6x - 8 = 0$

(Half the b value, so $6 \div 2 = 3$)

$(x + 3)^2 - 9 - 8 = 0$

(subtract this value squared as $(x + 3)^2$ multiplied out is $x^2 + 6x + 9$, not $x^2 + 6x$)

$(x + 3)^2 - 17 = 0$ (this is the completed the square form)

$(x + 3)^2 = 17$

$x + 3 = \pm\sqrt{17}$

$x = -3 \pm \sqrt{17}$

When written in the form $(x + p)^2 + r = 0$, you can determine key features of the graph.

The equation of the **line of symmetry** of the curve is $x = -p$

The co-ordinate of the **turning point** of the curve (minimum/maximum point) is $(-p, r)$

Solving quadratic inequalities

Solving a quadratic inequality is very similar to solving a quadratic equation.

Step 1: Solve the equation to find the critical values.

Step 2: Sketch the curve

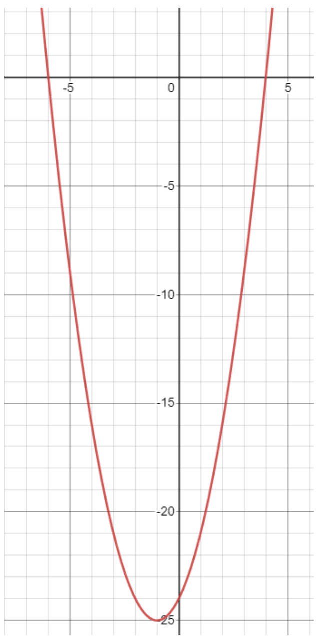
Step 3: Write down the appropriate inequality/inequalities

e.g. Solve $x^2 + 10x - 24 < 0$

Start by solving: $x^2 + 2x - 24 = 0$

$(x + 6)(x - 4) = 0$

$x = -6$ and $x = 4$ (these are the critical values)



The curve is a positive quadratic so is a 'u' shaped parabola.

The roots of the equation are at $x = -6$ and $x = 4$, so this is where it crosses the x axis.

The curve is < 0 (below the x-axis) when it is between $x = -6$ and $x = 4$.

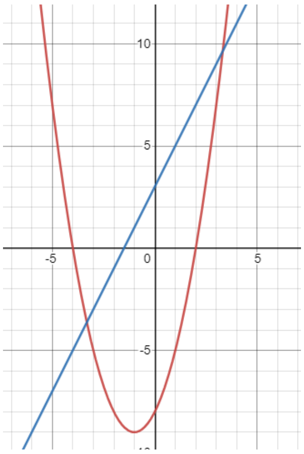
Therefore the solution to $x^2 + 10x - 24 < 0$ is:

$$-6 < x < 4$$

Note: if the question instead was solve $x^2 + 10x - 24 > 0$ we now need the sections above the x-axis which are not connected and so the solution would have been

$$x < -6 \text{ and } x > 4$$

Simultaneous equations where one is non-linear



As a non-linear graph will curve, the solution to simultaneous equations with a non-linear equation can have more than 1 answer.

If we are solving a quadratic and a linear graph there are either:

0 solutions – the graphs do not intersect

1 solution – the linear graph is a tangent to the curve and touches only once

2 solutions – the graph crosses twice (as shown on the left)

Solving the equations algebraically allows us to find the exact values of these intersections.

e.g. Solve $y = x^2 + 3x - 8$

$y = 2x + 3$

As both equations are y =, we can equate them

$x^2 + 3x - 8 = 2x + 3$

Rearrange so that one side = 0

$x^2 + x - 11 = 0$

This does not factorise so using the formula

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-11)}}{2 \times 1}$$

$x = 2.8541..$ and $x = -3.854...$

Substitute these back into $y = 2x + 3$

$y = 8.7082...$ and $y = -4.708...$

So the solutions are:

$x = 2.86$ and $y = 8.71$

$x = -3.85$ and $y = -4.71$

e.g. Solve $x^2 + y^2 = 10$ ← This is the equation of a circle

$y = 2x - 5$

This time we need to substitute $y = 2x - 5$ into the top equation.

$x^2 + (2x - 5)^2 = 10$

Multiply out the bracket

$x^2 + 4x^2 - 20x + 25 = 10$

Simplify and set one side = 0

$5x^2 - 20x + 15 = 0$

Factorise and solve

$5(x^2 - 4x + 3) = 0$

$5(x - 3)(x - 1) = 0$

$x = 3$ and $x = 1$

Substitute back into $y = 2x - 5$

When $x = 3$, $y = 1$

when $x = 1$, $y = -3$

Solutions need to be given in pairs with the correct x and y values matched up.

GLUE

HERE