Y10 Maths Knowledge Organiser Foundation Tier: Right Angled Triangles

What must I be able to do?

New content:

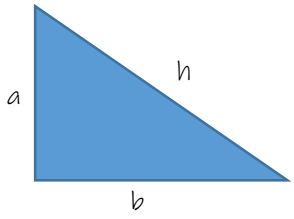
- ☐ Use Pythagoras' theorem to find a missing side in a right angled triangle
  - ➢ Sparx U385
- ☐ Use Pythagoras' theorem to solve problems
  - ➢ Sparx U541
- ☐ Use the three trigonometric ratios to find a missing side
  - ➢ Sparx U283
- ☐ Use the trig ratios to calculate an angle
  - ➢ Sparx U545
- ☐ Solve practical problems using trigonometry, including bearings and angles of elevation and depression
  - ➢ Sparx U967, U164
- ☐ Know certain values for exact trig functions
  - ➢ Sparx U627

Key vocabulary

Hypotenuse	The longest side of a right angled triangle. It is the side opposite the right angle.
Angle of elevation	The angle made with the ground by looking up at something.
Angle of depression	The angle made with the ground by looking down at something e.g. from the top of a cliff or tower.

Pythagoras' Theorem

Pythagoras' theorem states that in a right angled triangle, **the square of the hypotenuse is equal to the sum of the squares of the other two sides.**



$h^2 = a^2 + b^2$

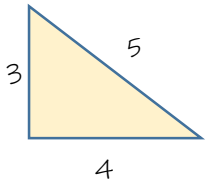
so therefore by rearranging we also get:

$a^2 = h^2 - b^2$ and

$b^2 = h^2 - a^2$

Pythagorean Triples

These are sets of 3 integer values which form a right angled triangle



The most common Pythagorean triple is the **3, 4, 5** triangle

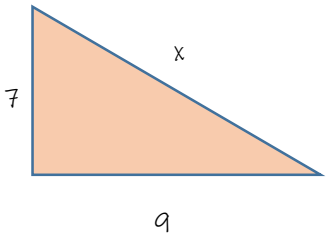
Any integer scale factor enlargement of a Pythagorean triple also gives another triple

e.g. 3, 4, 5 can become 6, 8, 10 (s.f. 2) or 9, 12, 15 (s.f. 3)

The next 6 primitive (non enlarged) Pythagorean triples are:

5, 12, 13	9, 40, 41
7, 24, 25	11, 60, 61
8, 15, 17	12, 35, 37

Example of Pythagoras



To find x we need to use Pythagoras's theorem as we know 2 sides and want to find the third.

In this question x is the hypotenuse as it is opposite the right angle in the triangle.

So our formula $h^2 = a^2 + b^2$ becomes

$$x^2 = 7^2 + 9^2$$

Note that it does not matter which is a and which is b out of the 7 and 9.

$$x^2 = 49 + 81$$

$$x^2 = 130$$

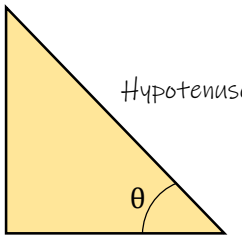
$$x = \sqrt{130}$$

$$x = 11.40175425$$

$$x = 11.4 \text{ (rounded to 1 decimal place)}$$

Trigonometric Ratios

For any right angled triangle, if we identify one angle we can label the 3 sides as shown



The ratio of each pair of the 3 sides, is always the same answer for a given size of the angle θ, regardless of the actual lengths of the sides.

This leads to the following definitions:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

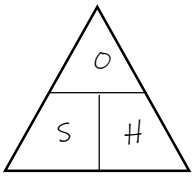
Sin is short for sine, cos for cosine and tan for tangent.

One way to remember these is the mnemonic **SOHCAHTOA** which gives each of the 3 ratios by their first letter.

We can also represent these ratios using formula triangles. In each case the letter in the middle goes at the top of the triangle

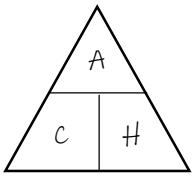
SOH

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$



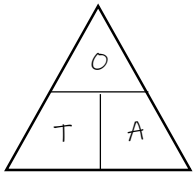
CAH

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

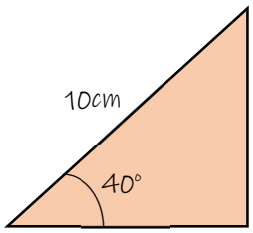


TOA

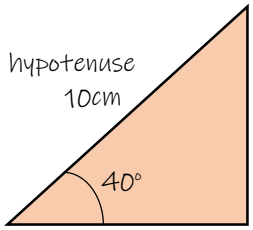
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$



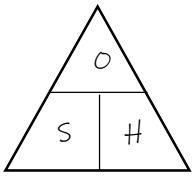
Using trigonometry to find a missing side



Start by labelling the two sides in the question



The ratio with O and H in is sine

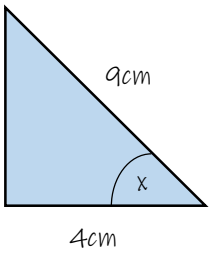


$$\sin 40 = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{x}{10}$$

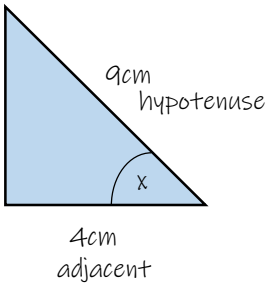
$$10 \times \sin 40 = x$$

$$x = 6.43 \text{ (2.d.p)}$$

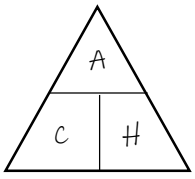
Using trigonometry to find a missing angle



Start by labelling the two sides in the question



The ratio with A and H in is cosine



$$\cos x = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{9}$$

$$x = \cos^{-1}\left(\frac{4}{9}\right)$$

$$x = 63.6°$$

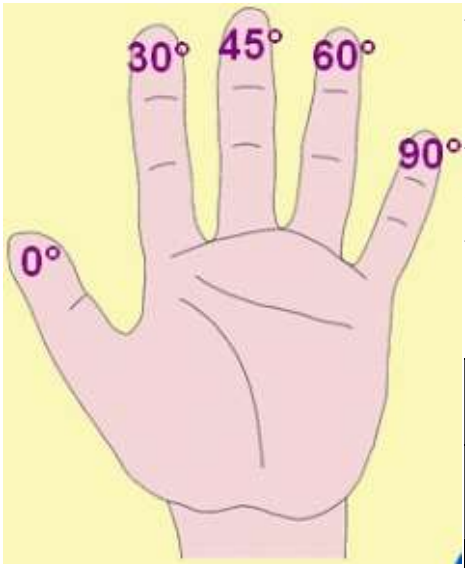
The **inverse** of each trig function is written as

$\sin^{-1} x$ $\cos^{-1} x$ and $\tan^{-1} x$

Use these when finding an angle

Exact trig values for 0, 30, 45, 60 and 90°

On a **non-calculator** paper you can be asked to complete a trigonometry question if the angle is 0, 30, 45, 60 or 90°. Therefore you need to learn the following standard values for these angles.



To find **sine** of one of these 5 angles, identify the correct finger on your left hand. Square root the number of fingers held up to the **left** of that finger and then divide by 2 to get an exact value for the sine of that angle.

The **cosine** is the square root of the number of fingers to the **right** of that finger and then divide by 2.

The **tangent** is the square root of the fraction of the number of fingers to the left (sine), divided by the number of fingers to the right (cosine).

	0	30	45	60	90
sin	$\frac{\sqrt{0}}{2}=0$	$\frac{\sqrt{1}}{2}=\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}=\frac{2}{2}=1$
cos	$\frac{\sqrt{4}}{2}=\frac{2}{2}=1$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}=\frac{1}{2}$	$\frac{\sqrt{0}}{2}=0$
tan	$\sqrt{\frac{0}{4}}=0$	$\sqrt{\frac{1}{3}}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$	$\sqrt{\frac{2}{2}}=\frac{\sqrt{2}}{\sqrt{2}}=1$	$\sqrt{\frac{3}{1}}=\frac{\sqrt{3}}{\sqrt{1}}=\sqrt{3}$	Does not exist

GLUE

HERE