Y10 Maths Knowledge Organiser Higher Tier: Algebraic Fractions, Functions and Proof

What must I be able to do?	Key vocabulary	
New content: ☐ Simplify algebraic fractions ➢ Sparx U437, U824, U103, U294, U685, U457 ☐ Solve equations involving algebraic fractions ☐ Find the output of a function ➢ Sparx U637 ☐ Find the inverse of a function ➢ Sparx U996 ☐ Find the composite of two functions ➢ Sparx U895, U448 ☐ Find an approximate solution for an equation using iteration ➢ Sparx U168, U434 ☐ Prove a given result using algebra ➢ Sparx U582	**Composite function**	A function which is the combination of two or more other functions. The notation seen would be fg(x) or similar.
	Iteration	Repeatedly applying a function in order to improve an answer. The answer to one is used as the input for the next iteration.
	Proof	Showing that a statement is always true or false, regardless of what value is used.

Simplifying algebraic fractions

These follow the usual rules for fractions if you are asked to add/subtract/multiply/divide. Common denominators will be needed for addition/subtraction.

A common question involves just a single fraction and being asked to simplify – the 1st step is to factorise both numerator and denominator, then cancel common factors.

e.g. Simplify $\frac{x^2+5x+6}{x^2-4}$

Factorising gives $\frac{(x+2)(x+3)}{(x+2)(x-2)}$ and as both the numerator and denominator are multiplied by (x + 2) these can be cancelled

Final answer: $\frac{x+3}{x-2}$

Note that the x's do not cancel on this final answer as the numerator and denominator are **not** being **multiplied** by x.

Solving algebraic fractions

These can involve multiple fractions, potentially leading to needing to solve a quadratic equation.

e.g. $\frac{3}{x-1} - \frac{2}{x+1} = 1$

A common denominator is (x – 1)(x + 1) so multiply both fractions by the other denominator

$$\frac{3(x+1)}{(x-1)(x+1)} - \frac{2(x-1)}{(x-1)(x+1)} = 1$$

Combine into a single fraction

$$\frac{3(x+1) - 2(x-1)}{(x-1)(x+1)} = 1$$

Multiply both sides by (x – 1)(x + 1)

$$3(x+1) - 2(x-1) = (x-1)(x+1)$$

Now expand, simplify and solve.

This particular quadratic factorises, but you may need to use the quadratic formula instead.

$$3x + 3 - 2x + 2 = x^2 - 1$$

$0 = x^2 - x - 6$ $\quad$ $0 = (x-3)(x+2)$ $\quad$ $x = 3, x = -2$

Substituting into functions

e.g. $f(x) = 3x - 4$

Find $f(2)$

This means substitute 2 into the function. Anywhere there is an x in the function, it is replaced with a 2 and then work it out.

$$f(2) = 3(2) - 4 = 6 - 4 = 2$$

Inverse functions

An inverse function does the opposite of a function. For a given function, if you substitute in one value and get an output, putting that output into the inverse function would find the original value. The notation for inverse function is $f^{-1}(x)$

e.g. $f(x) = 3x + 4$

Find $f^{-1}(x)$

Start by writing it as $y = 3x + 4$

Switch x and y around

$$x = 3y + 4$$

Now make y the subject of the formula

$$x - 4 = 3y$$

$$\frac{x - 4}{3} = y$$

The inverse function $f^{-1}(x) = \frac{x - 4}{3}$

Composite functions

A composite function is where 2 or more functions are applied in succession.

e.g. $h(x) = fg(x)$

What this means is substitute the value of $g(x)$ into the $f(x)$ function.

e.g. $f(x) = 3x + 4$, $g(x) = 2x + 5$

Find $fg(3)$

Working in to out, do $g(3)$ first.

$$g(3) = 2 \times 3 + 5 = 11$$

$$f(11) = 3 \times 11 + 4 = 37 \quad \text{So } fg(3) = 37$$

This can also be done algebraically.

e.g. Find $fg(x)$

Replace x in $f(x)$ with $g(x)$ and simplify

$$fg(x) = 3(2x + 5) + 4$$

$$= 6x + 15 + 4 = 6x + 19$$

Iteration

Iteration will often use function notation and the subscript of n and n + 1 to refer to the current value and the next value.

e.g. Find the a_2 a_3 and a_4 using the iteration formula $f_{n+1} = f_n + 3$ and $a_1 = 4$.

Using 4 as f_n and substituting gives us $a_2 = 4 + 3 = 7$.

On the next iteration use 7 as f_n so $a_3 = 7 + 3 = 10$

On the next iteration use 10 as f_n so $a_4 = 10 + 3 = 13$

Iteration is often used to find approximate solutions to an equation. Each additional iteration gives a more accurate solution.

Proof

Algebraic proof is a method of proving a statement to be true for all values. Do not just substitute some numbers in to show it works as this only proves it works for those particular values.

Some key algebraic terms that you need to know:

Property	**Algebraic notation**
Any number	n
Even number	$2n$
Odd number	$2n + 1$
Consecutive numbers	n , $n + 1$, $n + 2$, ...
Consecutive even numbers	$2n$, $2n + 2$, $2n + 4$, ...
Consecutive odd numbers	$2n + 1$, $2n + 3$, $2n + 5$, ...
2 numbers	n , m
Prove even	Can take a factor of 2 out of the expression
Prove a multiple of 3	Can take a factor of 3 out of the expression
Prove odd	Can show that there is an even number plus an odd number

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